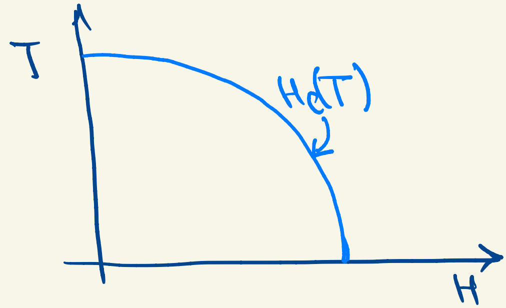
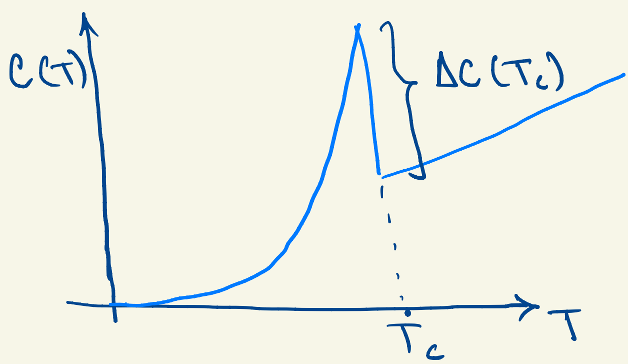
Thermodynamical constraints

Recall that the phase diagram of type-I SC looks like:



Further, as discussed earlier, the specific heat at $H=0$ jumps as a fn of T:



Interestingly, basic thermodynamical considerations relate ΔC to the curve $H_c(T)$.

The free energy $F(T, B)$ satisfies,

$$dF = -S\,dT + \frac{\vec{H} \cdot d\vec{B}}{4\pi}$$

where $\vec{H} = \vec{B} - 4\pi \vec{M}$.

From an experimental point of view, one may want to define a different free energy that is a function of T and H: $G = F - \frac{\vec{H} \cdot \vec{B}}{4\pi}$

$$\Rightarrow \quad dG = -S\,dT - \frac{\vec{B} \cdot d\vec{H}}{4\pi}$$

First consider the normal metal phase (i.e., no superconductivity). In this phase, $\vec{B} \approx \vec{H}$ (permittivity $\mu \sim 1$) $\Rightarrow \int_0^H dG\big|_T = -\int_0^H \frac{\vec{H} \cdot d\vec{H}}{4\pi}$

$$\Rightarrow \quad G_n(T, H) - G_n(T, H=0) = \frac{-H^2}{8\pi}.$$

On the other hand, in the SC phase,

$\vec{B} = 0 \Rightarrow G_s(T, H) - G_s(T, H=0) = 0.$

At $H = H_c$, $G_s(T, H_c) = G_n(T, H_c)$

$$\Rightarrow G_s(T, H=0) = G_n(T, H=0) - \frac{H_c^2(T)}{8\pi}$$

The entropy and specific heat are given by $S = -\frac{\partial G}{\partial T}$ and $C = T\frac{\partial S}{\partial T}$.

$$\Rightarrow S_s(T_c^-) - S_n(T_c^+) = -\frac{\partial}{\partial T}[G_s - G_n]$$

$$= \frac{1}{8\pi} \frac{\partial}{\partial T} H_c^2(T) \Big|_{T=T_c}.$$

$$= \frac{1}{4\pi} H_c \frac{dH_c}{dT} \Big|_{T=T_c}$$

Similarly, $C_s(T_c^-) - C_n(T_c^+)$

$$= \frac{T}{8\pi} \frac{\partial^2}{\partial T^2} H_c^2(T) \Big|_{T=T_c}$$

Recall that the Landau-Ginzburg free energy at $H=0$ is,

$$g = \int d^d x \left[r\,|\psi|^2 + u\,|\psi|^4 + |\nabla\psi|^2 \right]$$

where again $r = a(T-T_c)$ with $a>0$.

In the normal phase, $\psi = 0$

$\Rightarrow \quad g_n = 0.$

In the SC phase, $|\psi|^2 \sim -\frac{r}{u}$

and $g_s \sim -\frac{r^2}{u} \sim -(T-T_c)^2.$

$\Rightarrow \quad g_s(T, H=0) - g_n(T, H=0)$

$\sim -(T-T_c)^2.$

Combining this result with the previously derived expressions,

$$-H_c^2(T) \sim -(T-T_c)^2 \Rightarrow H_c(T) \sim T_c - T.$$

$$S_s(T_c^-) - S_n(T_c^+) = -\frac{\partial}{\partial T}[G_s - G_n]$$

$$= \frac{1}{4\pi} H_c \frac{dH_c}{dT}\Big|_{T=T_c} = 0 \Rightarrow \text{No Latent heat!}$$

Similarly, $C_s(T_c^-) - C_n(T_c^+)$

$$= \frac{T}{8\pi} \frac{\partial^2}{\partial T^2} H_c^2(T)\Big|_{T=T_c}$$

$$\sim \frac{T_c}{8\pi} \frac{\partial^2 (T-T_c)^2}{\partial T^2}\Big|_{T=T_c} \neq 0.$$

$\Rightarrow$ Specific heat jumps across the transition with $C_s > C_n$.