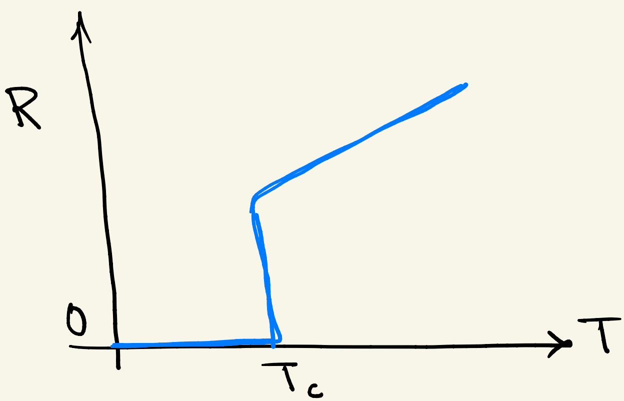
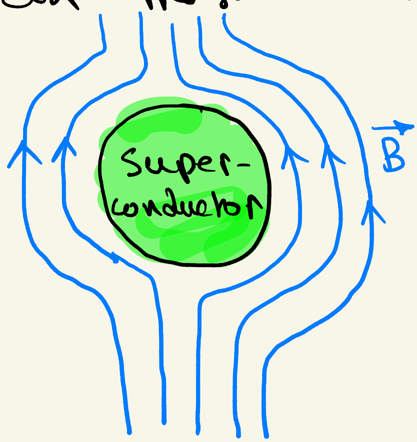
Phenomenology of superconductors

1. Zero electrical resistance below T_c.
(i.e. DC conductivity $= \infty$ below T_c).



2. Meissner effect (flux expulsion).

A superconductor expels magnetic field from its interior.



There are two qualitatively distinct kinds of superconductors, 'type I' and 'type II', that show different kind of response to external magnetic field.

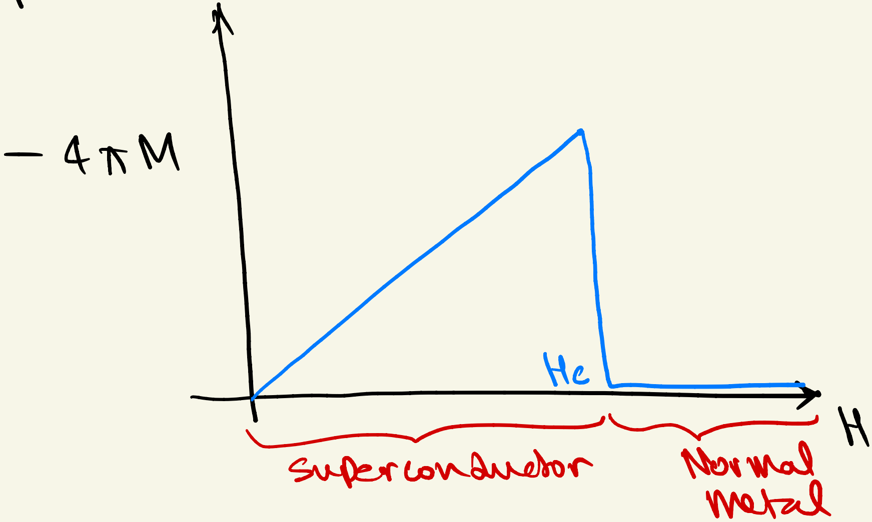
Type-I SC:

In type-I SC, magnetic field is completely expelled in the superconducting state. Denoting the applied field as $\vec{H}$ and the internal magnetization as $\vec{M}$, recall that the 'total' (i.e. actual) magnetic field $\vec{B}$ at a point $\vec{r}$ is $\vec{B} = \vec{H} + 4\pi\vec{M}$.

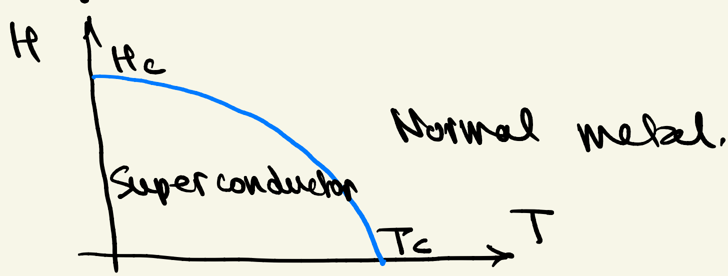
In a type-I SC, $\vec{B} = \vec{H} + 4\pi\vec{M} = 0$

i.e. $-4\pi\vec{M} = \vec{H}$. Magnetization $\vec{M}$ can be measured, and $\vec{H}$ is already known (it is the externally applied magnetic field and equals $\vec{B}$ far away from the SC).

One finds the following behavior for all temperatures less than superconducting T_c:



That is, there exists a critical field H_c, above which the material ceases to be a SC. Heuristically, above H_c, the free energy cost associated with expelling the magnetic field inside the SC, exceeds the free energy gain due to the system being a SC. The T-H phase diagram looks like:

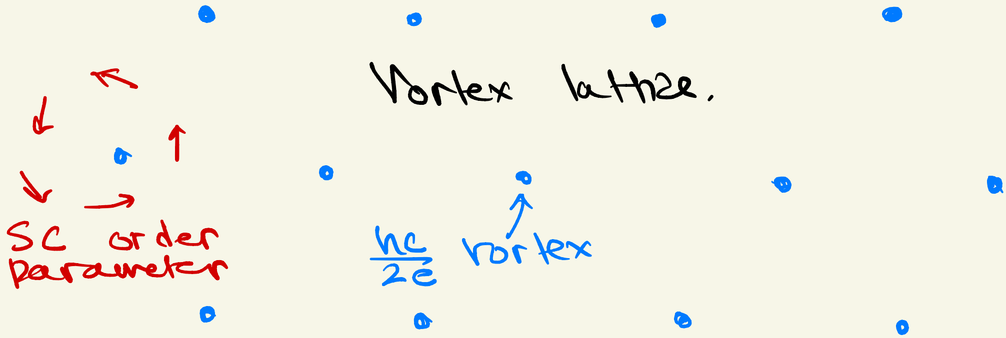


Type-II SC :

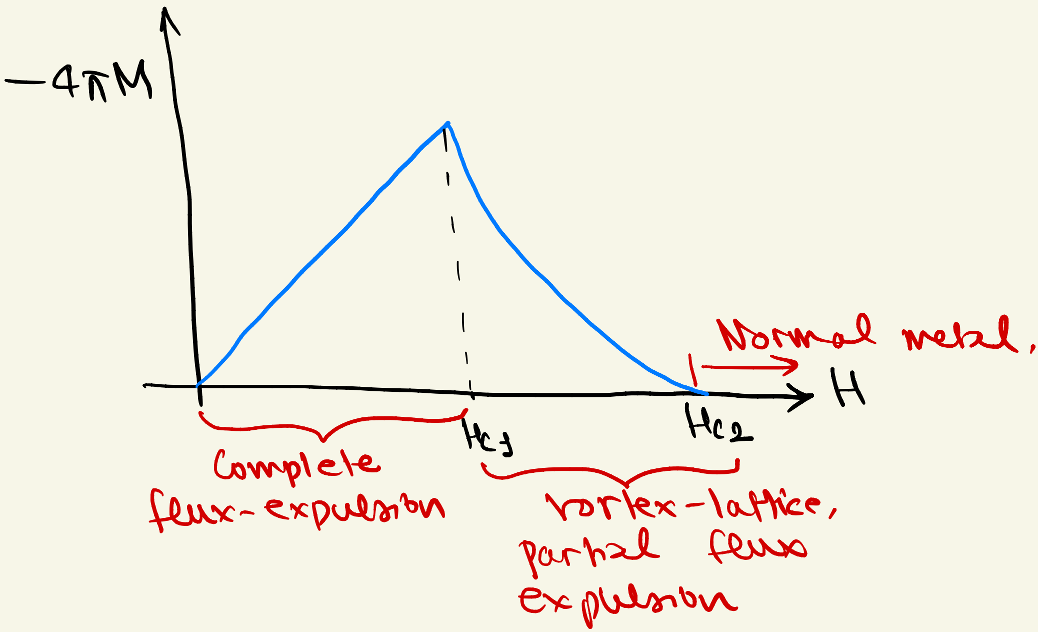
In a type-II SC, the transition from the complete flux-expulsion to no flux-expulsion is not abrupt. Instead, above a certain external field H_{c1}, magnetic field can enter the SC in the form of vortices in the SC order parameter. Each vortex traps $\frac{hc}{2e}$ magnetic flux.

At a higher field H_{c2}, the superconducting order parameter vanishes. In the regime $H_{c1} < H < H_{c2}$, one obtains a "vortex lattice superconducting phase", where a non-zero superconducting order-parameter exists everywhere in the material except the cores of the vortices, which form a lattice of their own.

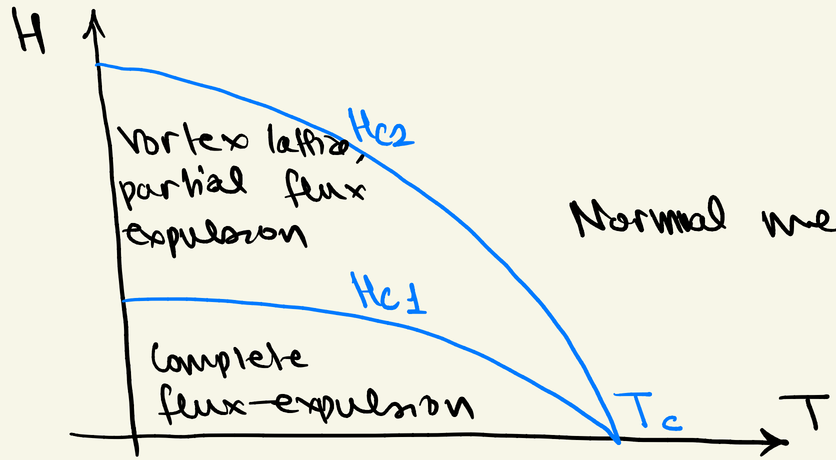
Vortex lattice.



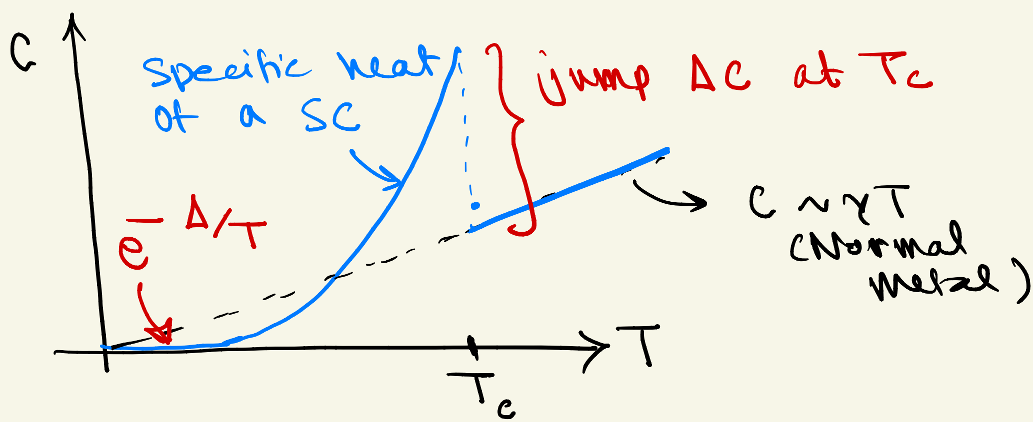
Due to partial meissner effect, one finds the following M vs H plot:



The H-T phase diagram looks like:

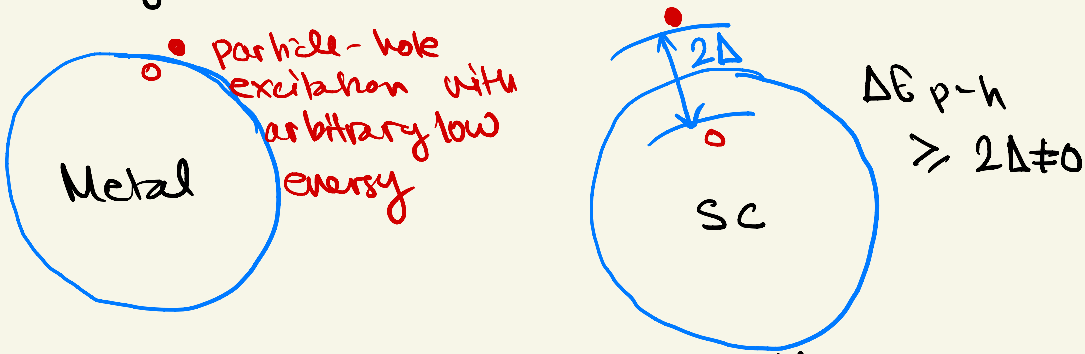


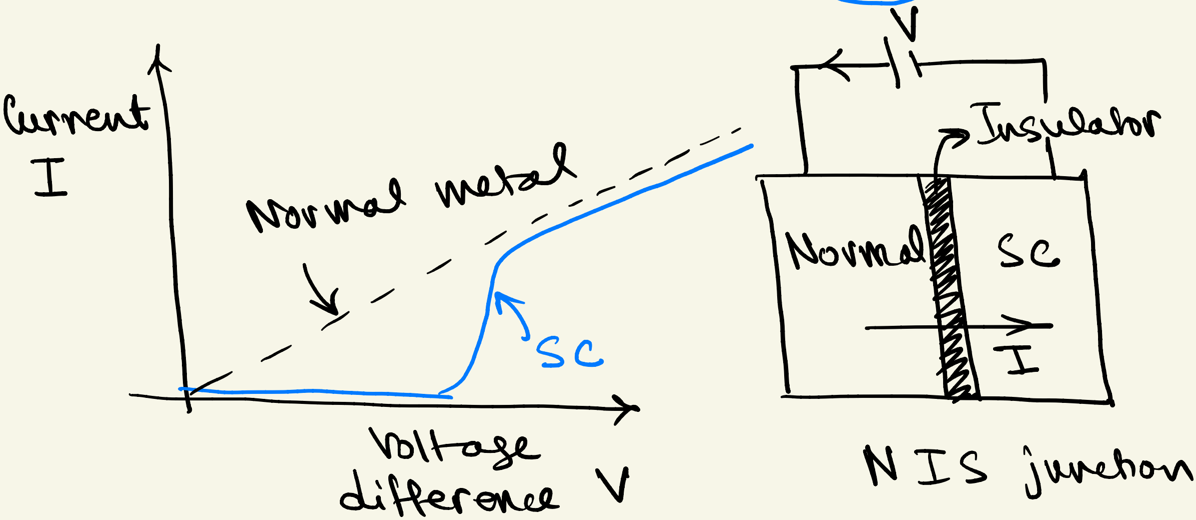
3. Specific heat: low-T behavior and singularity across the transition



entropy $S = \int \frac{C}{T} dT$. At low-T, an SC has lower entropy than a normal metal $\Rightarrow$ it is more ordered.

4. Gap to particle-hole excitations and relatedly, gap to single-e^- in tunneling experiments :





5. Josephson effect :

When two superconducting samples, with order parameter $\langle c^\dagger_\uparrow c^\dagger_\downarrow \rangle = e^{i\theta}$ that differs in the phase θ are placed next to each other, it leads to a flow of non-dissipative supercurrent $j \sim \sin(\nabla\theta)$ between them. Such a junction between two superconductors is called 'Josephson junction'.

In the presence of a DC voltage V, this current becomes $j \sim \sin(\nabla\theta - Vt)$ i.e. a DC voltage leads to an AC current.

6. Andreev reflection :

At an SC-metal junction, an electron incident at the interface with momentum $\vec{k}$ gets reflected as a hole with momentum $-\vec{k}$.