

Separability as a window into many-body mixed states

Tarun Grover
(UCSD)



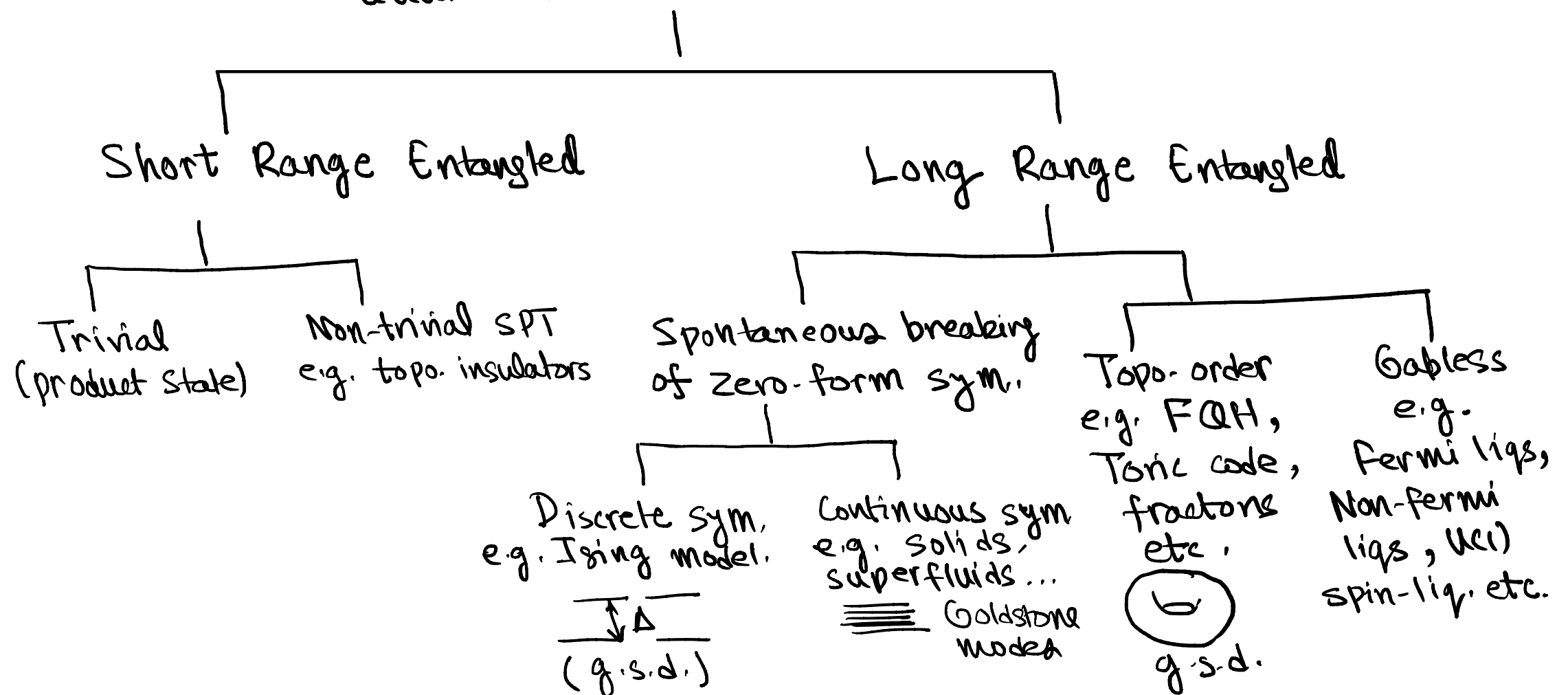
Yu-Hsueh Chen
(UCSD)

2309.11879 (PRL 132, 170602, 2024),
2310.07286, 2403.06553

Quantum Phases of Matter beyond pure states?

Lecture - 1 (Solid State Physics)

Quantum Phases of Matter ($T=0$)

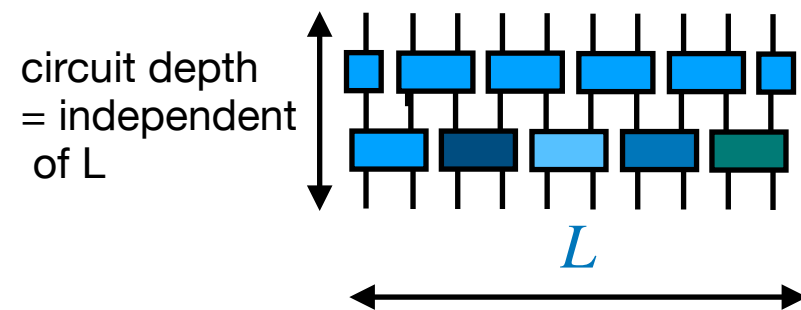


Quantum Phases of Matter beyond pure states?

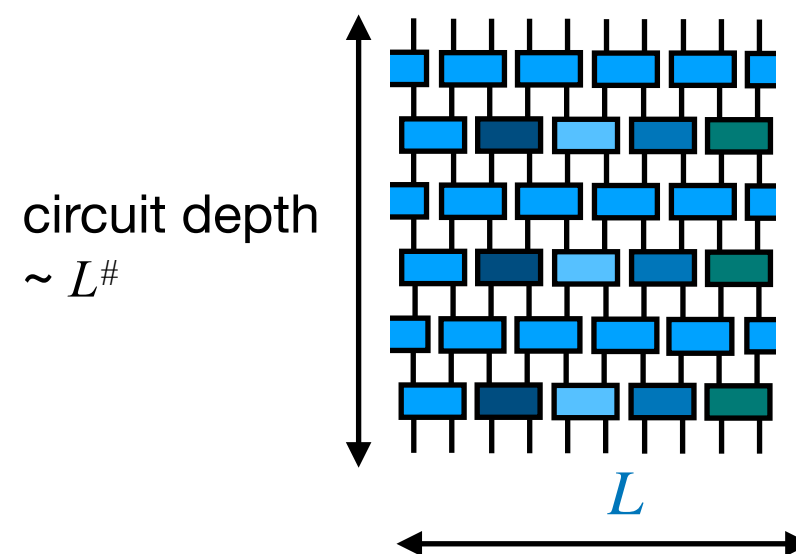
Lecture -1 (Solid State Physics)

Quantum Phases of Matter ($T=0$)

Short Range Entangled

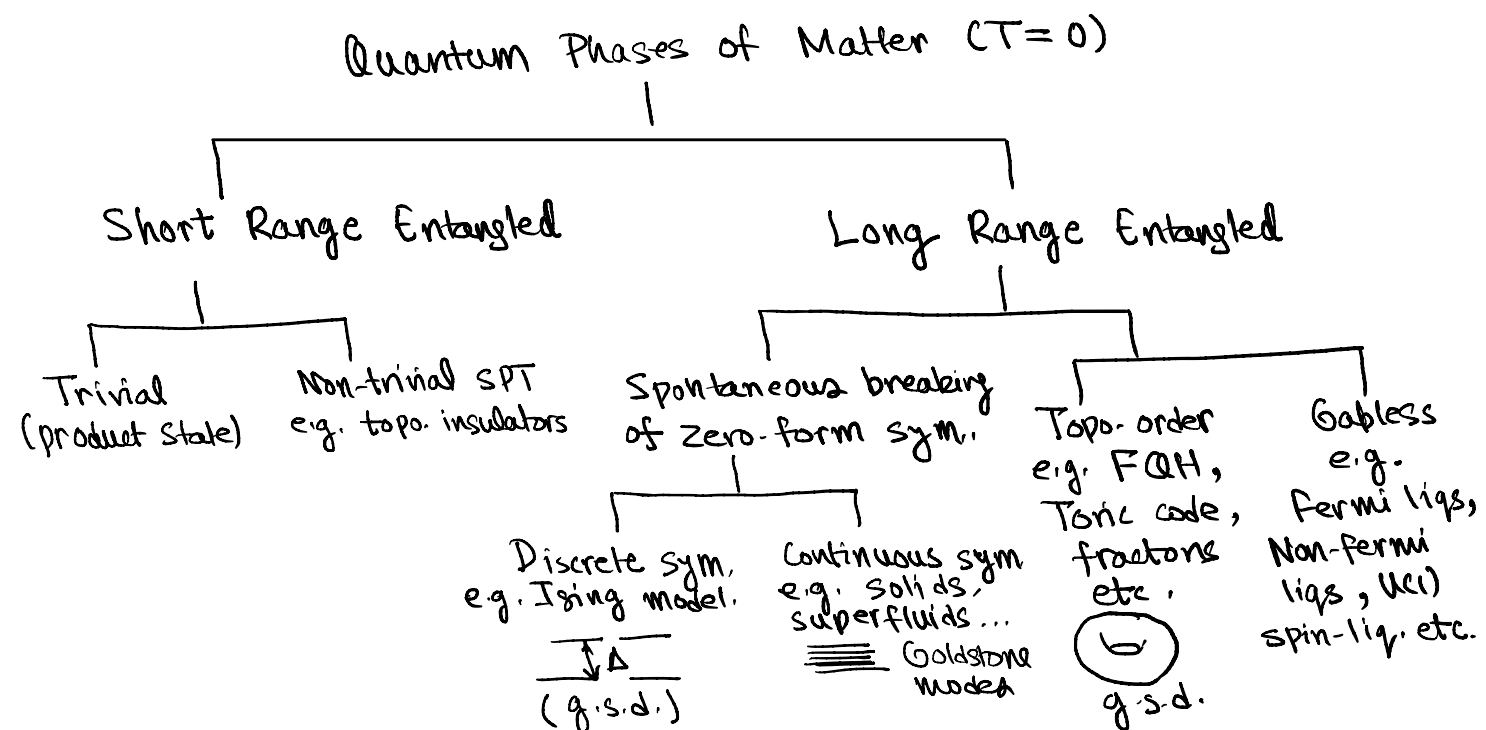


Long Range Entangled

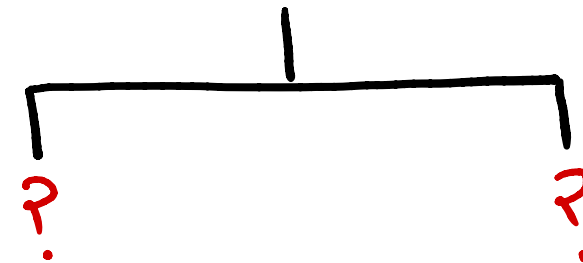


Quantum Phases of Matter beyond pure states?

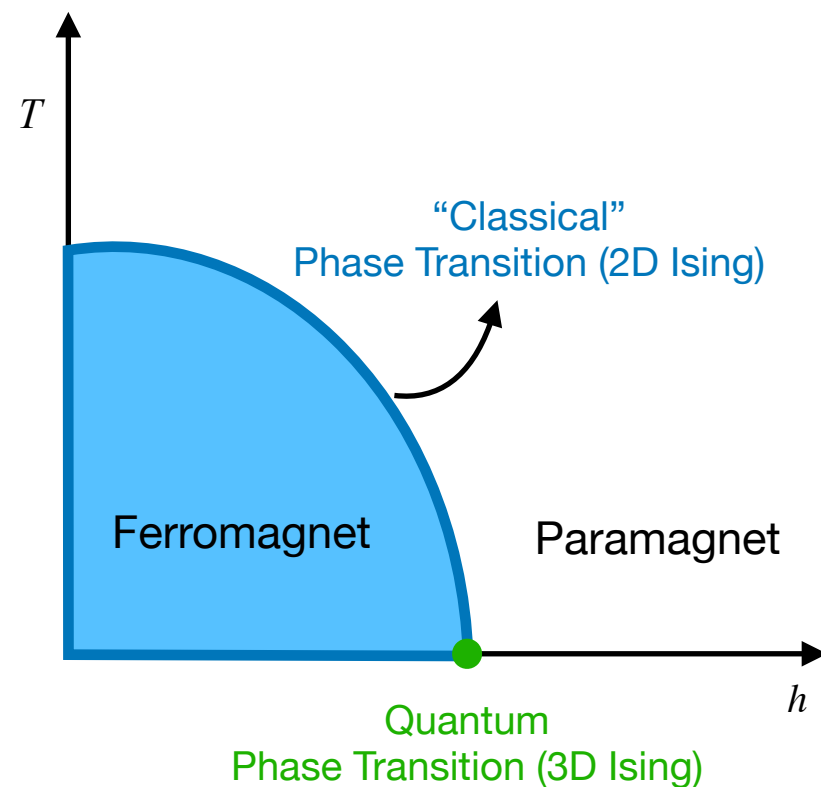
Lecture-1 (Solid State Physics)



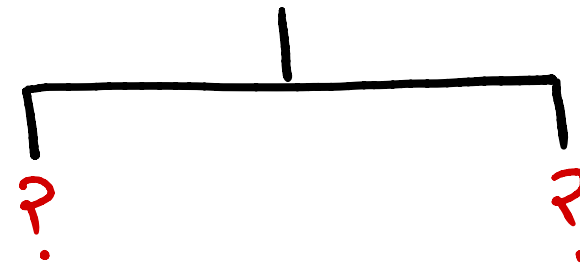
Mixed States



Quantum Phases of Matter beyond pure states?

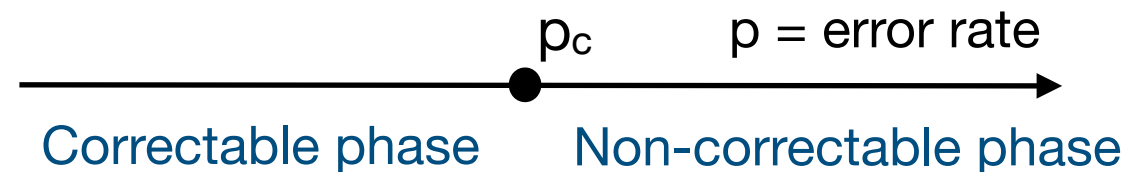


Mixed States

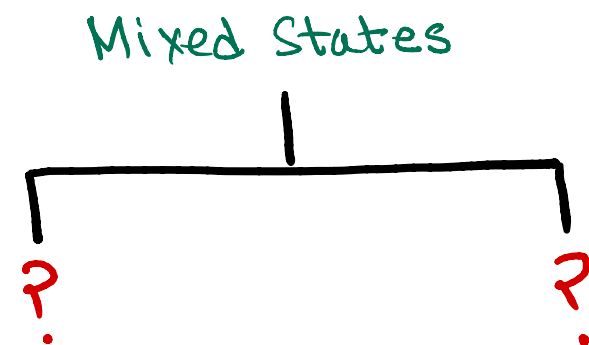


[Dennis, Kitaev,
Landahl, Preskill 2001]

Phase diagram of 2D toric code in the
presence of decoherence



Quantum Phases of Matter beyond pure states?



Several interesting developments:

[Equivalence between mixed-state phases](#) [Coser, Perez-Garcia '19; Rakovszky, Gopalakrishnan, Keyserlingk '23; Koenig, Pastawski '13; Hastings '11].

[Renormalization group approach and quantum error correction](#) [Sang, Zou, Hsieh '23; Sang, Hsieh '24; Lavasani, Vijay '24].

[Weak vs strong symmetries, corresponding SSB, and mixed-state SPTs](#) [de Groot, Turzillo, Schuch '22; Ma, Wang '22; Li, Jian, Xu '23; Ma et al '23, Lessa et al '24; Sala et al '24,...].

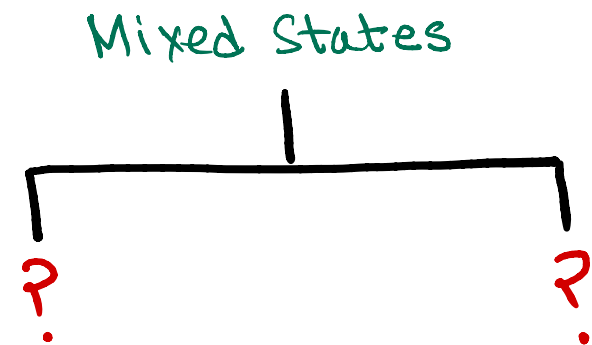
[Various entanglement measures](#) [Lu, Hsieh, TG '20; Fan, Bao, Altman, Vishwanath '23,...].

[Replica-based approach](#) [Bao, Fan, Altman, Vishwanath '23; Li, Jian, Xu '23; Zou, Sang, Hsieh '23, Li, Mong '24,...].

[Intrinsically mixed topological states, and higher-form symmetries](#) [Wang, Wu, Wang '23; Sohal, Prem '24; Ellison, Cheng '24; Li, Lee, Yoshida '24,...].

[LSM constraints/anomalies](#) [Kawabata, Sohal, Ryu '23; Zhou, Li, Li, Gu '23; Hsin, Luo, Sun '23; Lessa, Cheng, Wang '24; Wang, Li '24,...].

Quantum Phases of Matter beyond pure states?



In this talk we will employ a rather coarse characterization based on mixed-state entanglement, and discuss a few examples.

Zeroth Order question:

When is a mixed state unentangled (“separable”)?

Separable (= Unentangled) Mixed States

[Werner 1989] Iff a density matrix ρ admits a decomposition

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|, \text{ with } p_i > 0$$

where each $|\psi_i\rangle$ is unentangled between parties A and B i.e.

$|\psi_i\rangle = |\phi_{i,A}\rangle \otimes |\phi_{i,B}\rangle$, then ρ is bipartite separable (i.e. unentangled).

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Example: $\rho = p |\psi_{\text{Bell}}\rangle\langle\psi_{\text{Bell}}| + (1 - p) \frac{\mathbb{1}}{4}$

where $|\psi_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle |\downarrow\rangle - |\downarrow\rangle |\uparrow\rangle)$



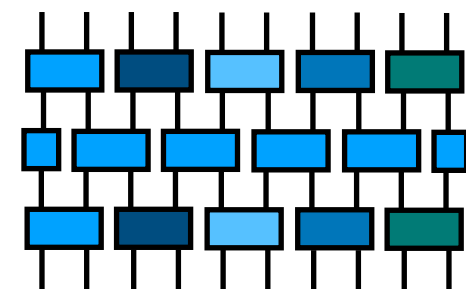
Many-body analogs?

Short-ranged entangled (SRE) mixed states = generalization of separability to many-body setup

If a density matrix admits a decomposition $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ where each $|\psi_i\rangle$ is short-ranged entangled (i.e. can be prepared via a finite-depth, local, unitary circuit), then we will call ρ a “short-ranged entangled (SRE) mixed-state”.

[Hastings 1106.6026]

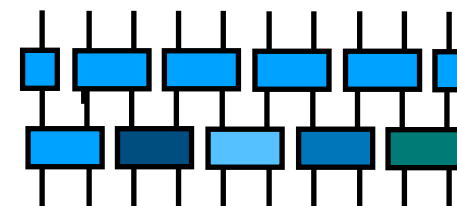
circuit depth
= constant
= L^0



L

$|\psi_1\rangle$

probability p_1



$|\psi_2\rangle$

probability p_2

Infinitely many decompositions of a density matrix into pure states (!)

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = \sum_i p'_i |\psi'_i\rangle\langle\psi'_i| = \sum_i p''_i |\psi''_i\rangle\langle\psi''_i| = \dots$$

No general algorithm to determine if it is SRE/LRE.

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No general algorithm to determine if it is SRE/LRE.

A few useful tools:

- Explicit demonstration that a density matrix is SRE (for simple models).
- Lieb-Robinson bounds to show LRE (= long-range entangled i.e. not SRE).
- Anomaly (e.g. Lieb-Schultz-Mattis) based arguments.
- (Heuristic) Mixed-state entanglement measures such as negativity.

Quantum entanglement vs classical long-range correlations

Coser, Perez-Garcia (1810.05092): Two mixed states in the same phase if they can be connected via finite time, local Lindbladian evolution.

Example: (a) $|00\dots 0\rangle\langle 00\dots 0|$ (b) $\frac{1}{2} (|00\dots 0\rangle\langle 00\dots 0| + |11\dots 1\rangle\langle 11\dots 1|)$

belong to different phases of matter due to long-range *classical* correlations in (b).

However, both states are unentangled, and hence “trivial” from separability perspective.

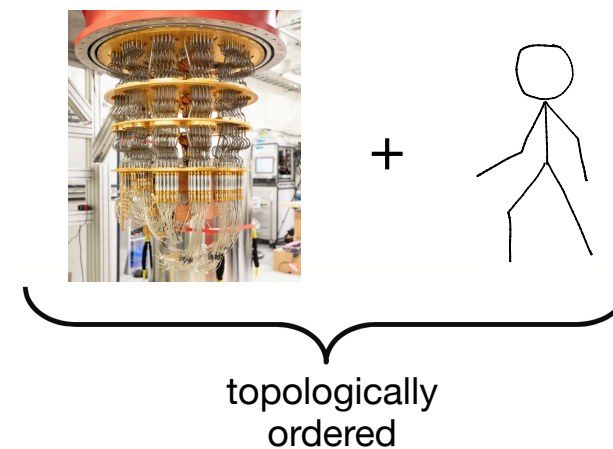
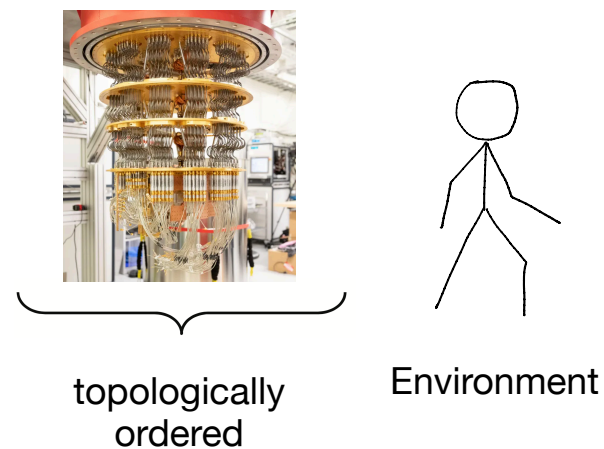
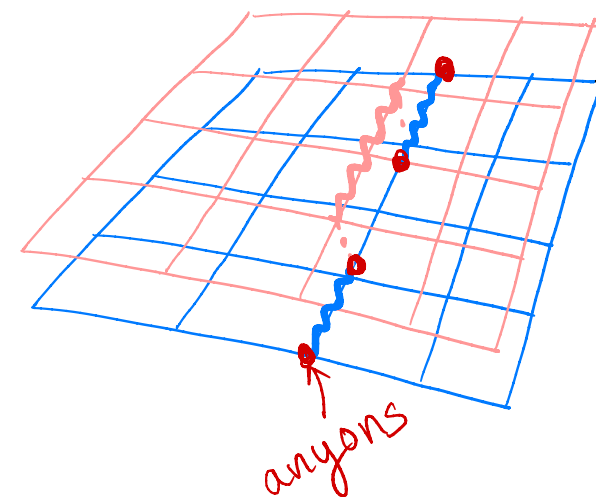
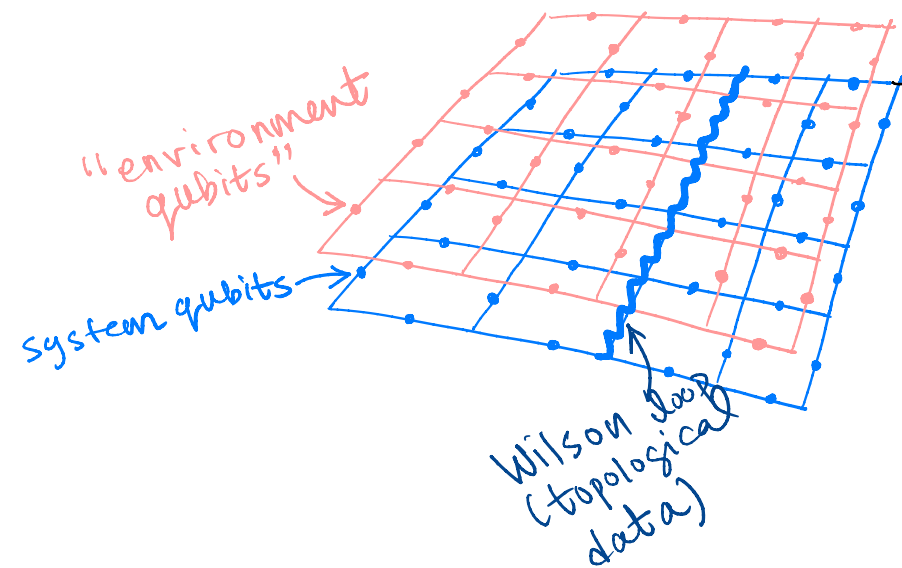
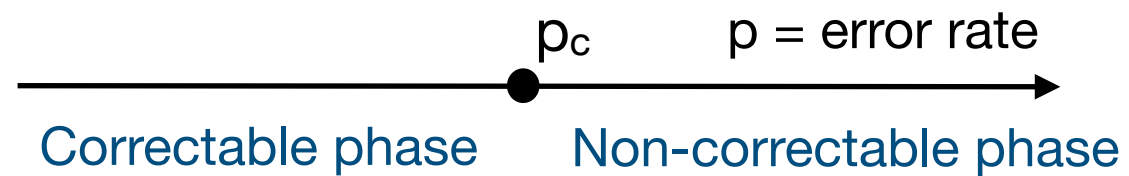
One may also define an SRE mixed state as one that has an SRE purification (e.g. Ma, Wang 2209.02723). In this definition, classical correlations will again be regarded as non-trivial (e.g. state (b) has no SRE purification).

Outline

- Decoherence induced separability transitions.
 - A. Topological ordered states.
 - B. SPT states.
 - C. Chiral states.
- Separability transitions in Gibbs states.

Decoding transition as a separability transition

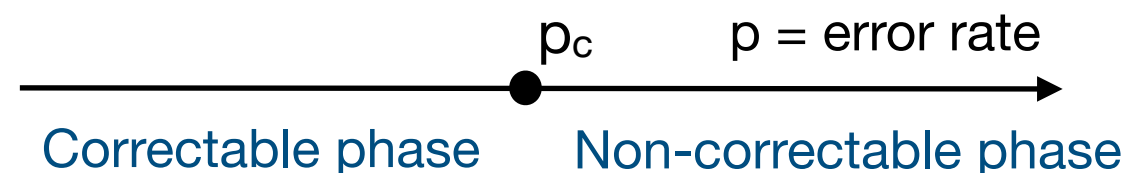
[Dennis, Kitaev,
Landahl, Preskill 2001]



Decoding transition as a separability transition

Recent works, in particular, Fan, Bao, Altman, Vishwanath [2301.05689; 2301.05687], and Lee, Jian, Xu [2301.05238] have formulated decoding transition as an intrinsic transition for the decohered mixed-state.

- Logical qubit lost to environment for $p \geq p_c$ (as detected via “coherent information”).
- Renyi negativity also shows a phase transition from $\log(2)$ to zero.
- “Markov length” diverges at $p = p_c$ [Sang, Hsieh 2024].



Can one show that the density matrix is SRE in the non-correctable phase?

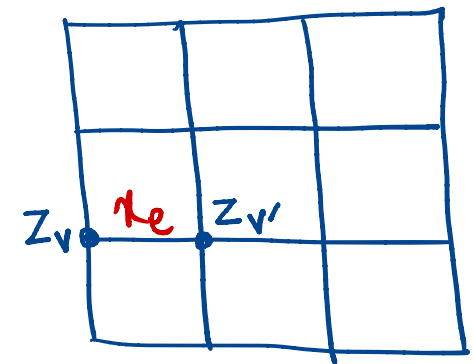
Decohered density matrix

$$H_{2d \text{ toric}} = - \sum_v \text{diagram of vertex } v \text{ with four blue edges labeled } z \text{ and } v \text{ in the center} - \sum_p \text{diagram of face } p \text{ with four red edges labeled } x \text{ and } p \text{ in the center}$$

local channel: $\mathcal{E}_e[\rho_0] = pZ_e\rho_0Z_e + (1-p)\rho_0$

[Dennis, Kitaev,
Landahl, Preskill '01]

$$\rho \propto \sum_{x_e} \mathcal{Z}_{2d \text{ Ising}, x_e} |\Omega_{x_e}\rangle \langle \Omega_{x_e}|$$



$$\mathcal{Z}_{2\text{d Ising}, x_e}(p) = \sum_{z_v} e^{\beta \sum_e x_e \prod_{v \in e} z_v} \quad \tanh(\beta) = 1 - 2p$$

$$|\Omega_{x_{\mathbf{e}}}\rangle \propto \prod_v (I + \prod_{e \ni v} Z_e) |x_{\mathbf{e}}\rangle = \text{subset of toric code eigenstates}$$

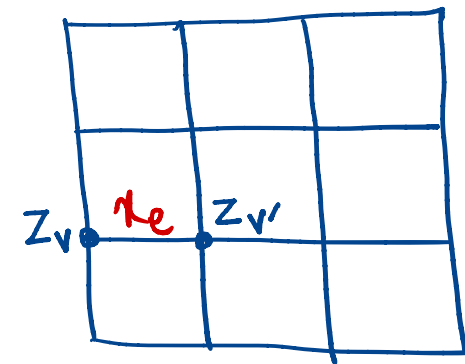
Decohered density matrix

$$H_{2d \text{ toric}} = - \sum_v \begin{array}{c} z \\ \hline v \\ \hline z \end{array} - \sum_p \begin{array}{c} X \\ \hline X \boxed{p} X \\ \hline X \end{array}$$

local channel: $\mathcal{E}_e[\rho_0] = p Z_e \rho_0 Z_e + (1 - p) \rho_0$

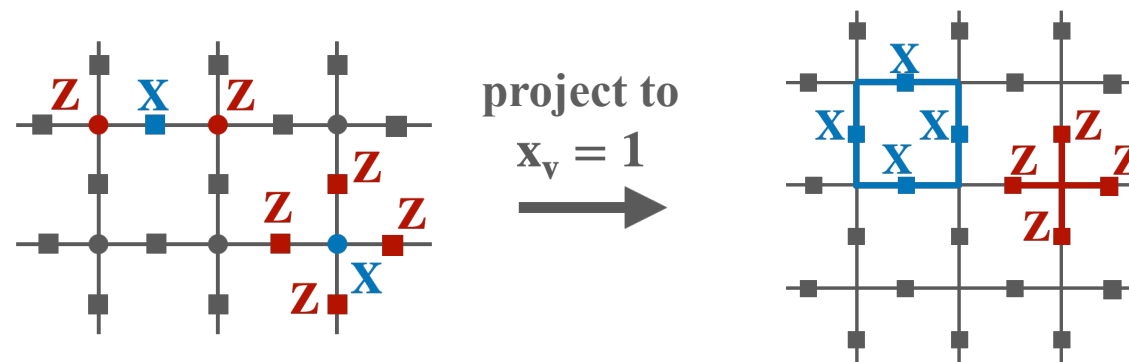
[Dennis, Kitaev,
Landahl, Preskill '01]

$$\rho \propto \sum_{x_e} \mathcal{Z}_{2d \text{ Ising}, x_e} |\Omega_{x_e}\rangle \langle \Omega_{x_e}|$$



Another viewpoint:

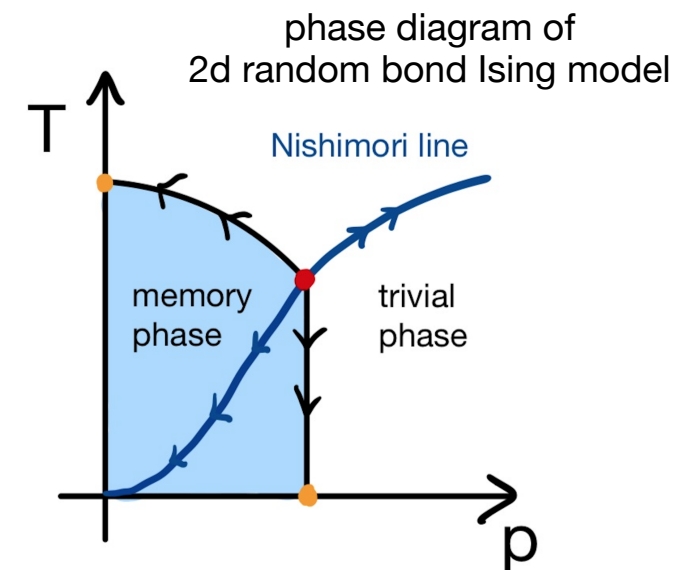
Statistical weights $\mathcal{Z}_{2d \text{ Ising}, x_e}$
inherited from “parent” cluster state.



“ $\sqrt{\rho}$ trick”

Key idea: Write decohered ρ as

$$\rho = \sum_{z_e} \underbrace{\sqrt{\rho}|z_e\rangle\langle z_e|\sqrt{\rho}}_{|\psi_m\rangle} \equiv \sum_m |\psi_m\rangle\langle\psi_m|$$



Claim: **All** $|\psi_m\rangle$ undergo transition from topological to trivial *precisely* at p_c corresponding to the decoding transition. Topological Renyi entanglement of $|\psi_m\rangle$, as well as tunneling probability from one logical state to another relates to free energy cost of inserting a domain wall in 2d random-bond Ising model along the Nishimori line.

Similar argument works for several other CSS codes in 2d and 3d, including fracton codes e.g. X-cube model.

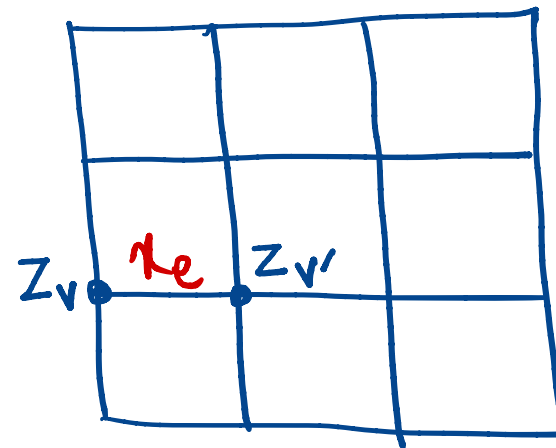
Structure of the “optimal” decomposition

$$\rho = \sum_{z_{\mathbf{e}}} \sqrt{\rho} |z_{\mathbf{e}}\rangle \langle z_{\mathbf{e}}| \sqrt{\rho}$$

$$\sqrt{\rho} |z_{\mathbf{e}} = 1\rangle \propto \sum_{x_{\mathbf{e}}} [\mathcal{Z}_{2\text{d Ising}, x_{\mathbf{e}}}(p)]^{1/2} |x_{\mathbf{e}}\rangle$$

$$\mathcal{Z}_{2\text{d Ising}, x_{\mathbf{e}}}(p) = \sum_{z_{\mathbf{v}}} e^{\beta \sum_e x_e \prod_{v \in e} z_v}$$

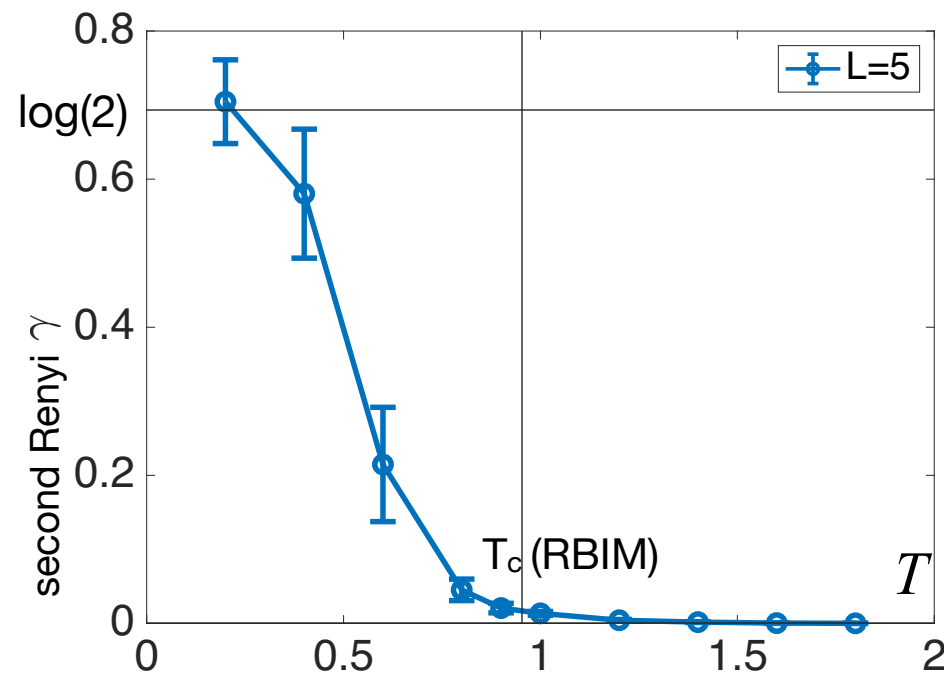
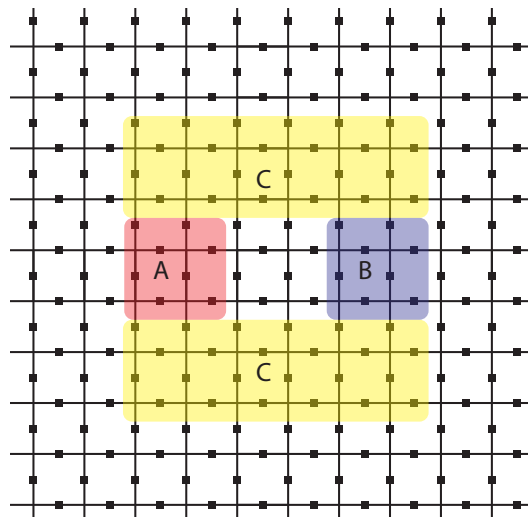
$$\tanh(\beta) = 1 - 2p$$



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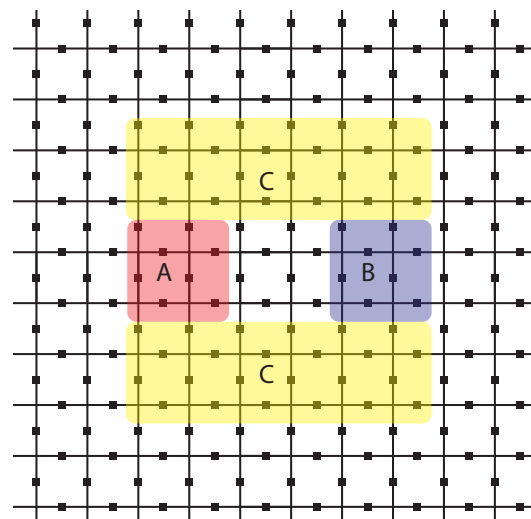
[Ting-Tung Wang, Menghan Song,
Zi Yang Meng (Unpublished)]

$(\tanh(1/T) = 1 - 2p)$

Structure of the “optimal” decomposition

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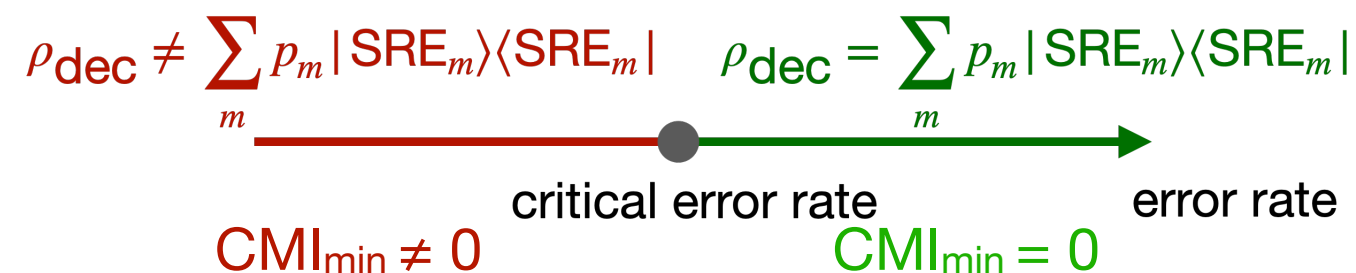


More generally, for a mixed-state ρ , define

$$\text{CMI}_{\min} = \inf \sum_i p_i I(A : B | C)_{\psi_i}$$

where the infimum is taken over all possible decompositions of ρ as $\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|$

$\text{CMI}_{\min}$ = “long-range part of mixed-state entanglement”



Separability perspective on double state & canonical purification

$$|\rho\rangle = \rho_{\mathcal{H}} \otimes I_{\bar{\mathcal{H}}} |\Phi\rangle_{\mathcal{H} \otimes \bar{\mathcal{H}}} \quad \text{“double state”} = \text{canonical purification of } \rho^2 / \text{tr}(\rho^2)$$

[e.g. Bao, Fan, Altman, Vishwanath 2023; Li, Jian, Xu 2023]

$$|\sqrt{\rho}\rangle = \sqrt{\rho}_{\mathcal{H}} \otimes I_{\bar{\mathcal{H}}} |\Phi\rangle_{\mathcal{H} \otimes \bar{\mathcal{H}}} \quad \text{canonical purification of } \rho$$

If $|\sqrt{\rho}\rangle$ is SRE, then $\rho \otimes 1$ can be written as a convex sum of SRE pure states.

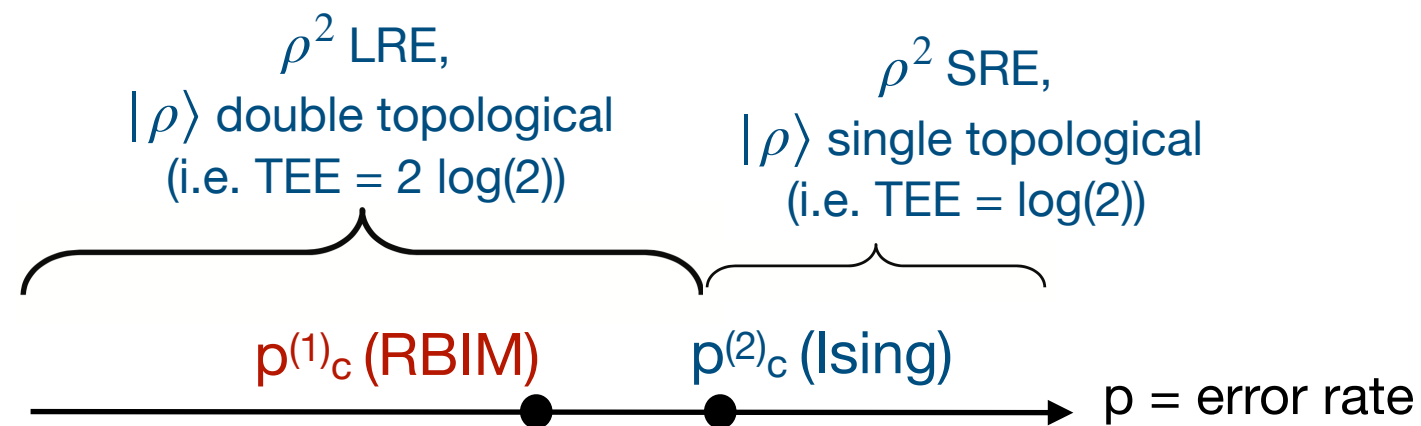
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Separability perspective on double state & canonical purification

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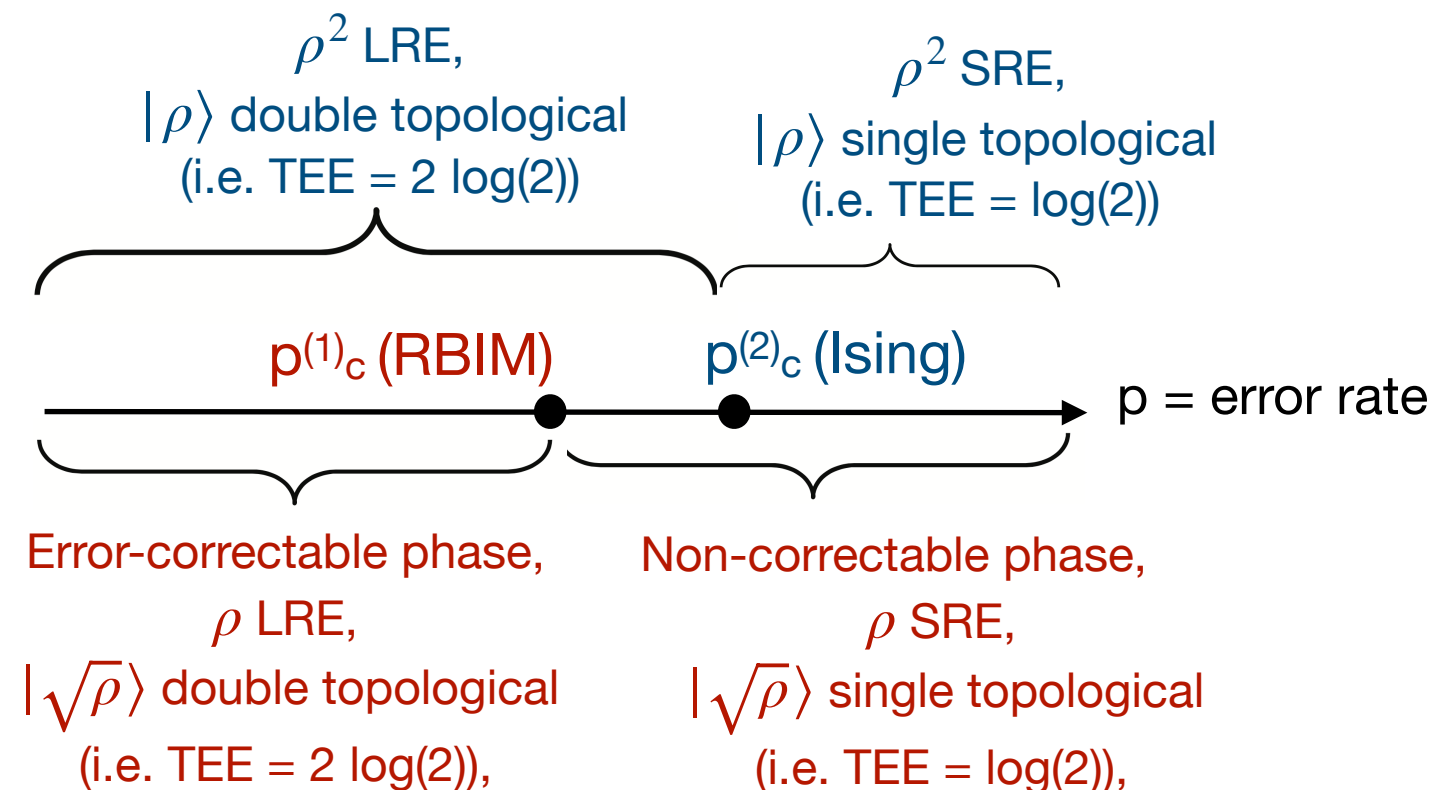


Separability perspective on double state & canonical purification

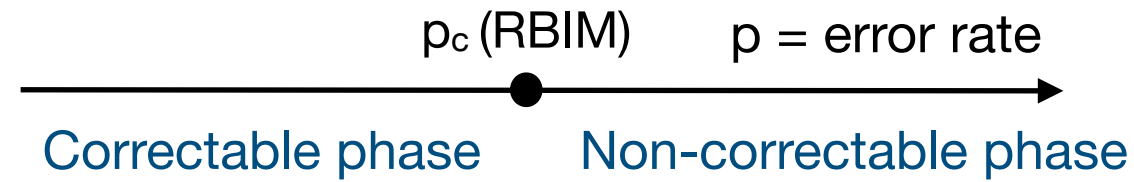
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Purification to a trivial state for $p \geq p_c$



$$\rho(\beta) = \text{tr}_A(|\Psi\rangle\langle\Psi|)$$

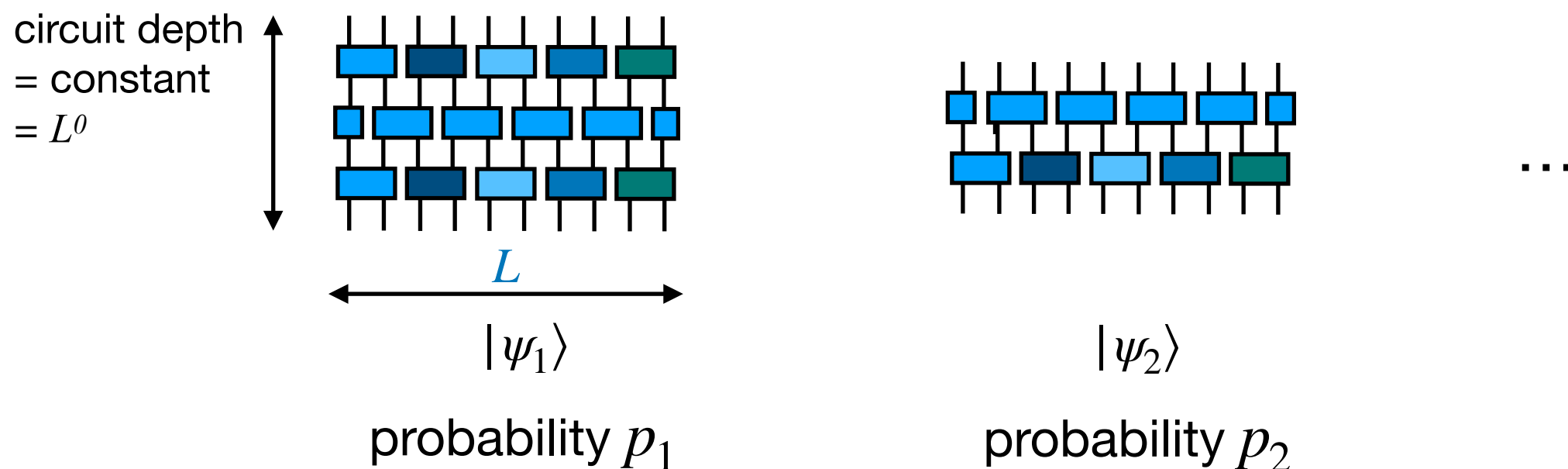
$$|\Psi\rangle = \left(\prod_p U_{p,A} \right) \left(|\psi(\beta)\rangle \otimes |0\rangle_A^{\otimes N_p} \right)$$

$$|\psi(\beta)\rangle = \sum_{x_e} \sqrt{Z_{x_e}(\beta)} |x_e\rangle \quad U_{p,A} = \frac{I}{\sqrt{2}} + i \frac{B_p \otimes X_A}{\sqrt{2}}$$

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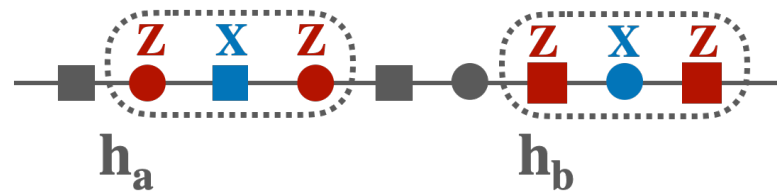
Incorporating symmetries

If a density matrix admits a decomposition $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ where each $|\psi_i\rangle$ is short-ranged entangled, and can be prepared via a finite-depth, local, unitary circuit composed of **symmetric** gates, then we will call ρ a “**sym-SRE mixed-state**”.



Each local gate  satisfies, $[\text{gate}, U] = 0$, where U is the generator of the symmetry.

Symmetry enforced separability transitions in cluster states



$$H = - \sum_{j=1}^N (Z_{b,j-1} X_{a,j} Z_{b,j} + Z_{a,j} X_{b,j} Z_{a,j+1})$$

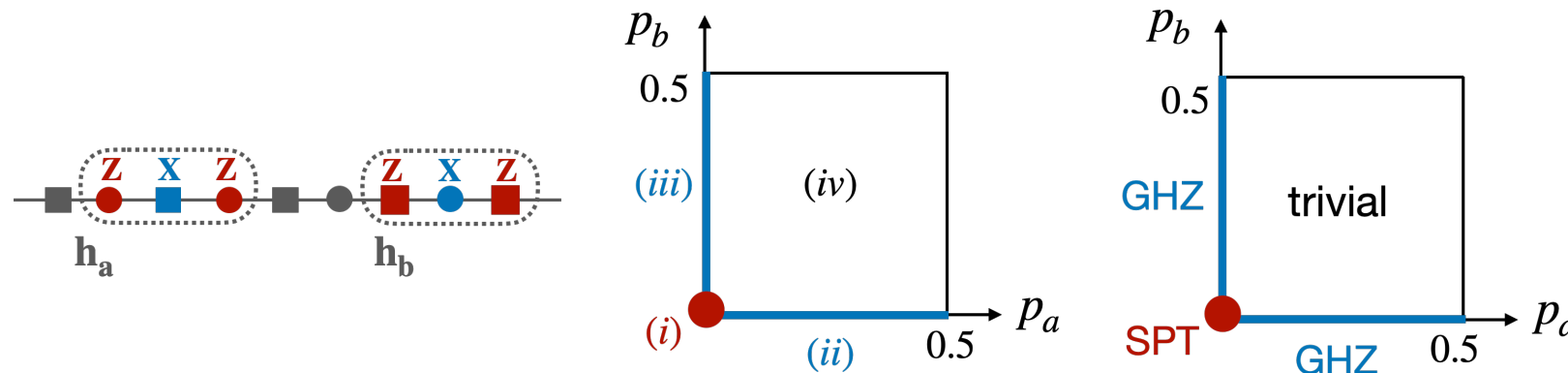
$$= \sum_{j=1}^N h_{a,j} + h_{b,j}$$

Ground state $\rho_0 = \prod_j (1 - h_{a,j})(1 - h_{b,j})$ is a non-trivial SPT phase (i.e. sym-LRE)
protected by $Z_2 \times Z_2$ symmetry.

Let's subject ρ_0 to the channel $\mathcal{E}_{a/b,j}[\rho] = (1 - p_{a/b})\rho + p_{a/b}Z_{a/b,j}\rho Z_{a/b,j}$

Is the resulting state sym-SRE at any non-zero p_a and/or p_b ?

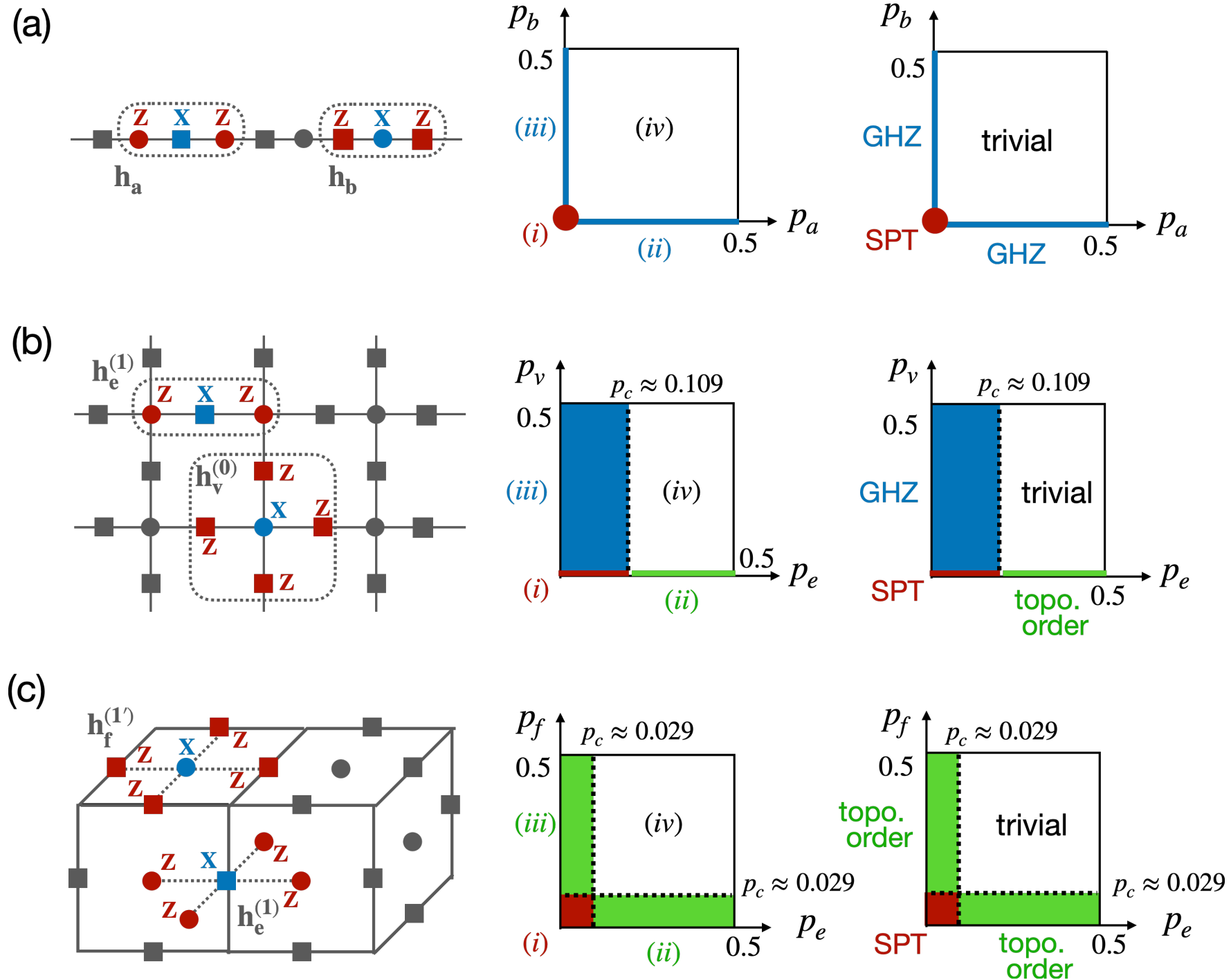
Symmetry enforced separability transitions in cluster states



Result: ρ sym-LRE as long as $p_a = 0$ or $p_b = 0$ (regions i, ii, iii).
 sym-SRE if both p_a, p_b non-zero (region iv). Proof uses
 Lieb-Robinson bound [Yu-Hsueh Chen, TG, 2310.07286].

Ma, Wang [2209.02723], and Ma et al [2305.16399]: in regions i, ii, iii, ρ cannot be purified to an SRE pure state using symmetric, finite-depth channel. Recent relation to SPT as a resource for transmitting quantum information: Zhang, Agarwal, Vijay [2405.05965].

Symmetry enforced separability transitions in cluster states



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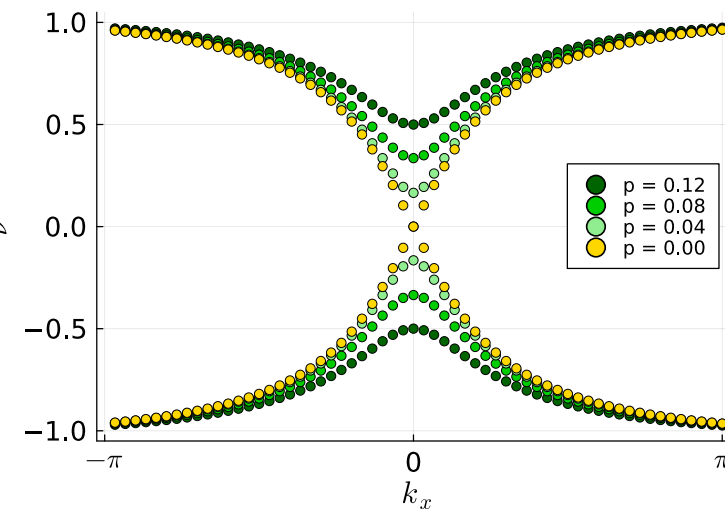
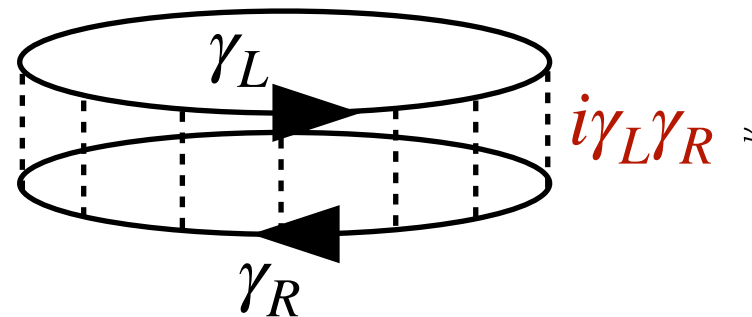
p+ip SC subjected to fermionic Kraus operators

$$\rho_d = \mathcal{E}[|p+ip\rangle\langle p-ip|]$$

$$\mathcal{E}_j[\rho] = (1 - p)\rho + p\gamma_j\rho\gamma_j$$

(explicitly breaks fermion parity from strong to weak)

Double state:



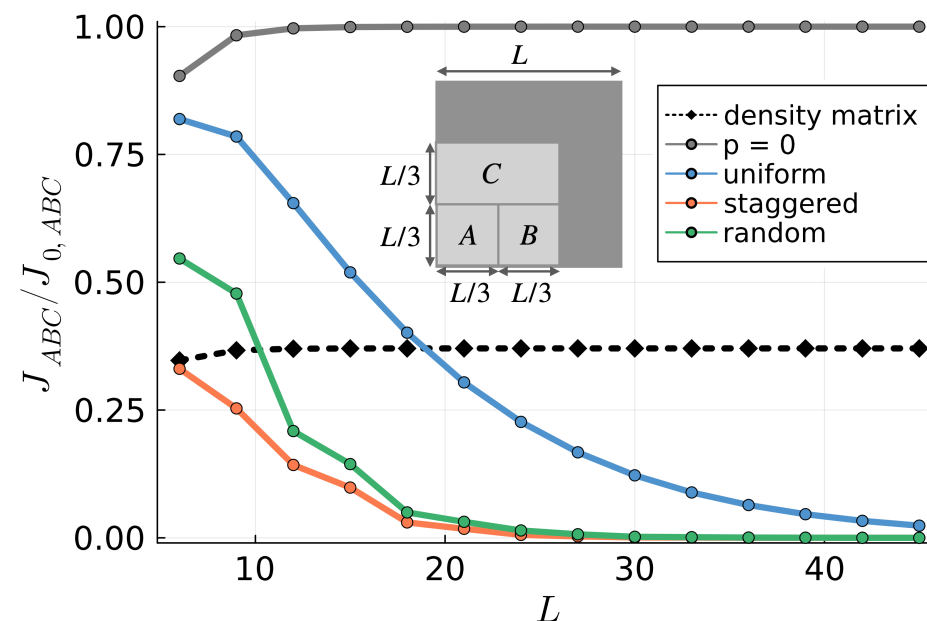
Claim:

$$\rho_d = \sum_i p_i |\text{Gapped non-chiral}\rangle_i \langle \text{Gapped non-chiral}|$$

basic idea:

$$\rho_d = \sum_m \sqrt{\rho_d} |\text{product state}\rangle_m \langle \text{product state}| \sqrt{\rho_d}$$

[Yu-Hsueh Chen, TG, 2310.07286]



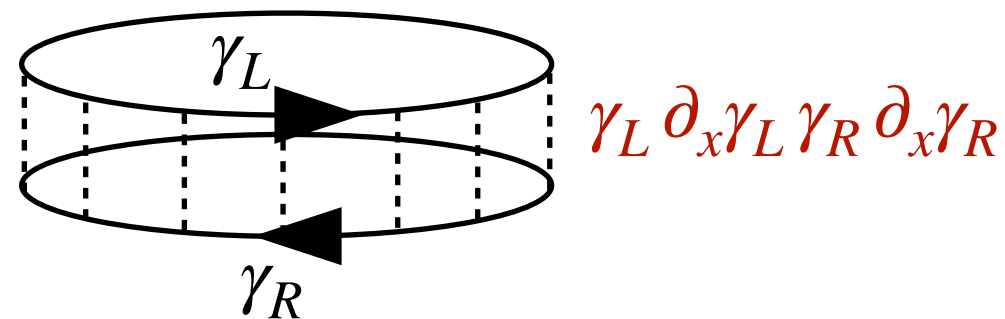
p+ip SC subjected to bilinear Kraus operators

$$\rho_d = \mathcal{E}[|p+ip\rangle\langle p-ip|]$$

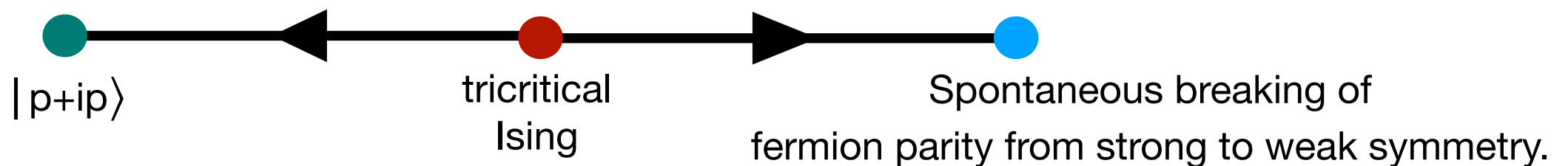
$$\mathcal{E}_{\langle \mathbf{x}, \mathbf{y} \rangle}[\rho] = (1 - p)\rho + p\gamma_{\mathbf{x}}\gamma_{\mathbf{y}}\rho\gamma_{\mathbf{x}}\gamma_{\mathbf{y}}$$

(fermion parity = strong symmetry)

Double state:



Field theory arguments suggest the following phase diagram for the double state:



[Yu-Hsueh Chen, TG, 2310.07286]

Is there a phase transition in “single copy”, as detected by, say, $S = -\text{tr}(\rho \log(\rho))$?

If yes, strong-to-weak symmetry breaking of fermion parity, no pure state analog.

Other examples of strong-to-weak SSB: [Ma, Wang '22; Li, Jian, Xu '23; Ma et al '23, Lessa et al '24; Sala et al '24]

- Decoherence induced separability transitions.
- Separability transitions in Gibbs states.

A. Quantum Ising model.

B. Toric codes.

C. NLTS Hamiltonians.

Spontaneous symmetry breaking as a separability transition

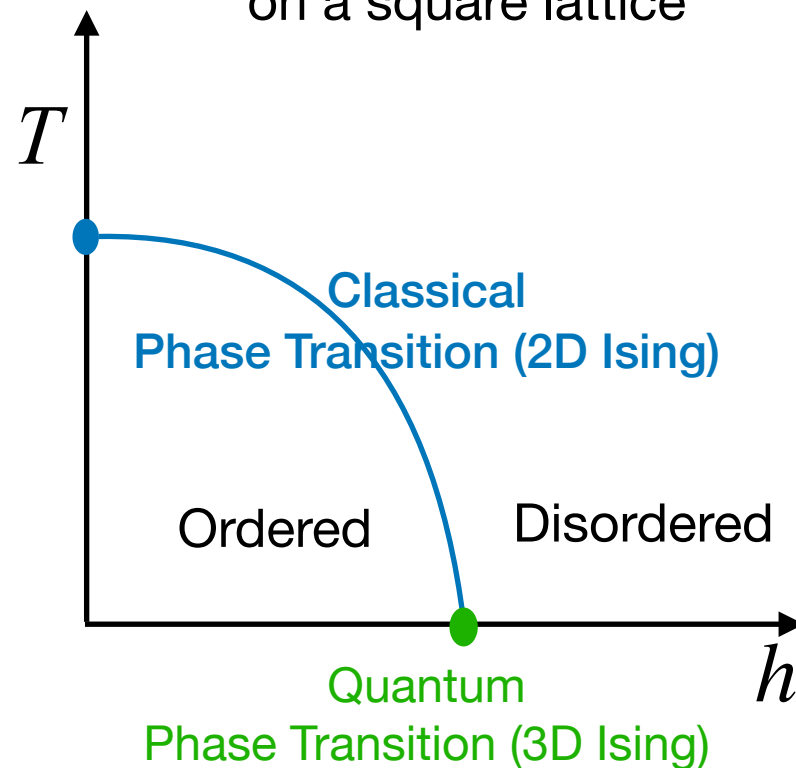
Claim:

$$H = - \sum_{\langle i,j \rangle} Z_i Z_j - h \sum_i X_i \quad \text{The Gibbs state } \rho \propto e^{-H/T} \text{ is sym-LRE for } T < T_c$$

on a square lattice

Proof by contradiction: Assume ρ is sym-SRE for $T < T_c$.

separate ρ into even and odd Ising sectors: $\rho = \rho_+ + \rho_-$



$$\rho_{\pm} = \sum_{\alpha} p_{\alpha, \pm} |\psi_{\alpha, \pm}\rangle \langle \psi_{\alpha, \pm}|$$

$$\rho \text{ sym-SRE} \Rightarrow |\psi_{\alpha, \pm}\rangle \text{ SRE}$$

$$\Rightarrow \langle \psi_{\alpha, \pm} | Z_j Z_k | \psi_{\alpha, \pm} \rangle - \langle \psi_{\alpha, \pm} | Z_j | \psi_{\alpha, \pm} \rangle \langle \psi_{\alpha, \pm} | Z_k | \psi_{\alpha, \pm} \rangle \sim e^{-|i-j|/\xi}$$

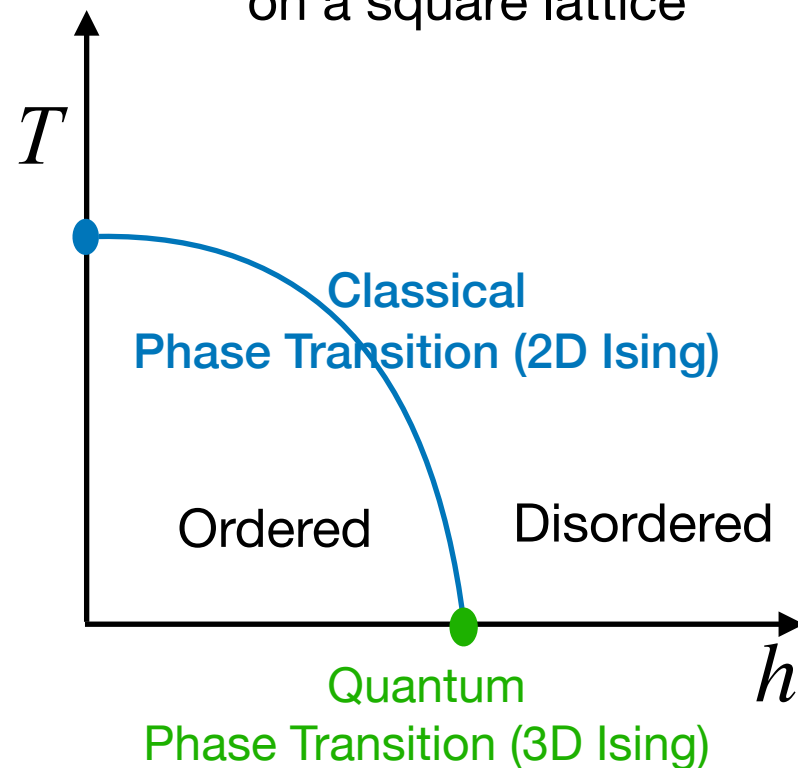
$$\Rightarrow \text{tr}(\rho Z_j Z_k) = \sum_{\pm} \sum_{\alpha} p_{\alpha, \pm} \langle \psi_{\alpha, \pm} | Z_j Z_k | \psi_{\alpha, \pm} \rangle \sim e^{-|i-j|/\xi}$$

Contradiction because of spontaneous long-range order for $T < T_c$

Spontaneous symmetry breaking as a separability transition

$$H = - \sum_{\langle i,j \rangle} Z_i Z_j - h \sum_i X_i$$

on a square lattice



“optimal” sym-SRE decomposition:

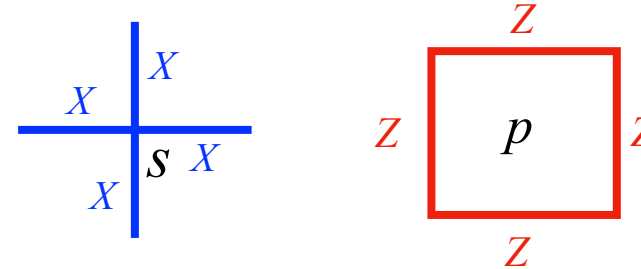
$$\rho = \sum_{x_{\mathbf{v}}} \sqrt{\rho} |x_{\mathbf{v}}\rangle \langle x_{\mathbf{v}}| \sqrt{\rho}$$

Conjecture: Pure states $\sqrt{\rho} |x_{\mathbf{v}}\rangle$ are SRE only for $T > T_c$.

- Decoherence induced separability transitions.
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Consider Gibbs state of Toric code in various dimensions...

$$H = -\lambda_A \sum_s A_s - \lambda_B \sum_p B_p$$

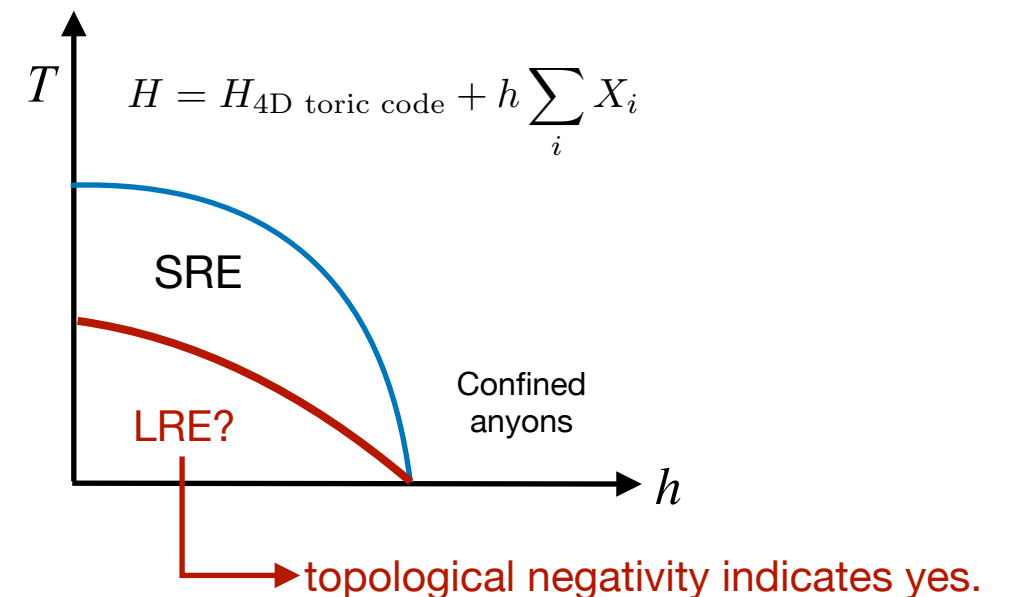
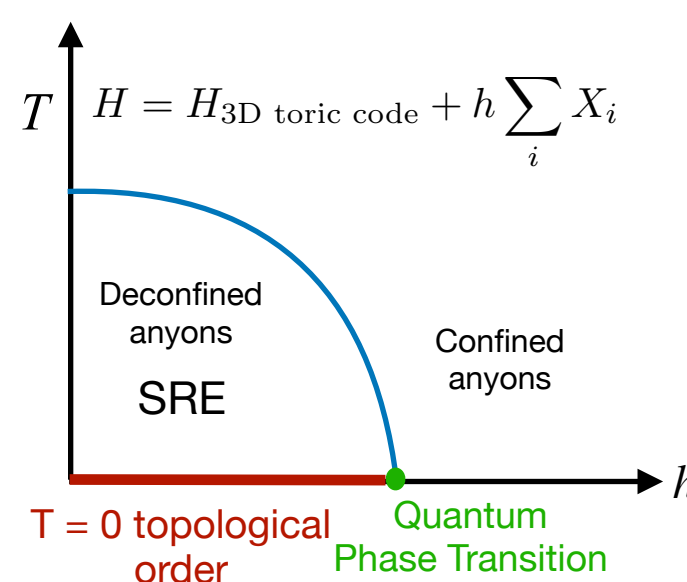


Let's write ρ as:
$$\rho = \frac{1}{Z} \sum_m \underbrace{e^{-\beta H/2} |m\rangle \langle m| e^{-\beta H/2}}_{= |\phi_m\rangle} = \frac{1}{Z} \sum_m |\phi_m\rangle \langle \phi_m|$$

where $\{ |m\rangle \}$ = complete set of product states in the X or Z basis.

One can argue that all $|\phi_m\rangle$ are SRE whenever $T > \min(T_A, T_B)$ where T_A, T_B correspond to the critical temperatures of the classical Hamiltonians A_s, B_p

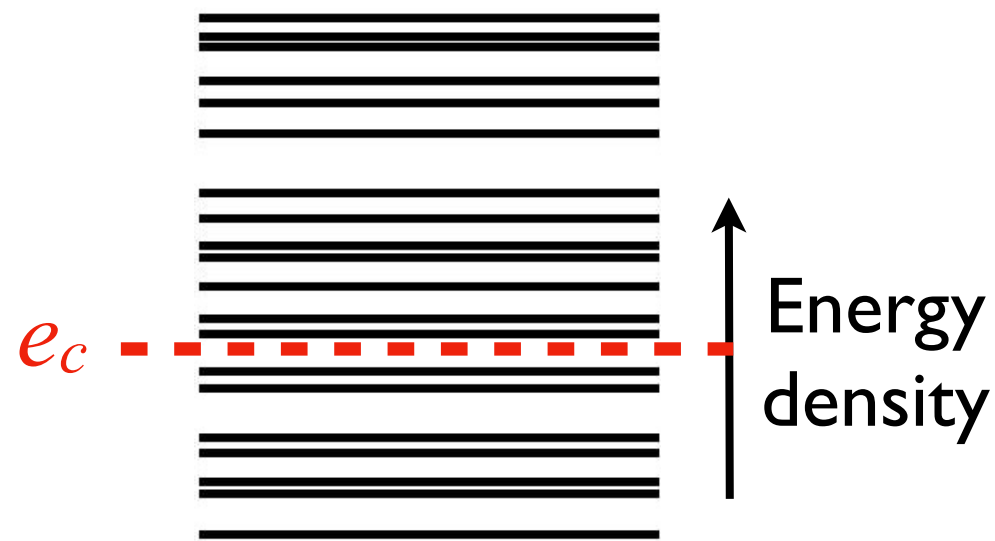
	T_A	T_B
2+1-D	0	0
3+1-D	0	λ_B
4+1-D	λ_A	λ_B



- Decoherence induced separability transitions.
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An exotic separability transition

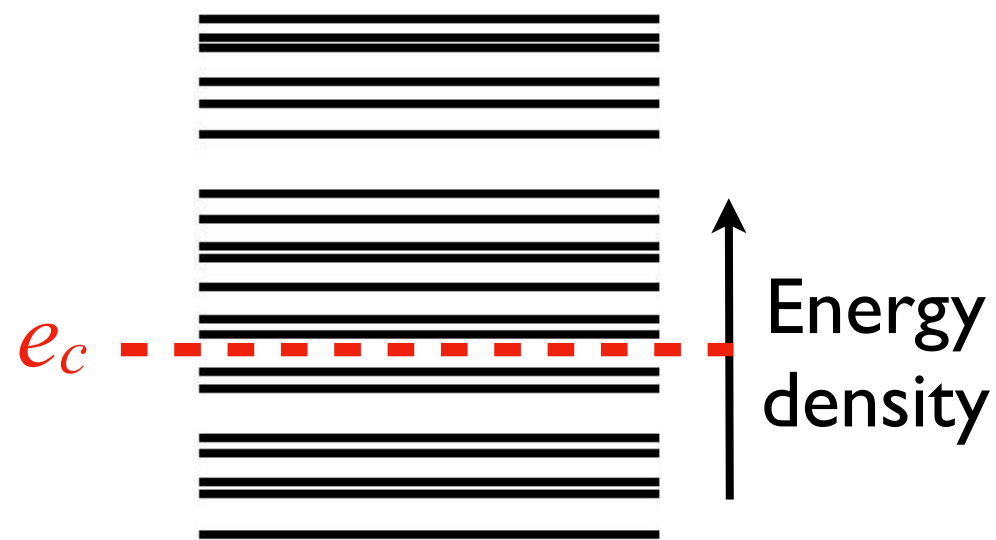
Recently, quantum Hamiltonians have been discovered [Panteleev, Kalachev 2022; Leverrier, Zemor 2022; Dinur et al 2022; Anshu, Breuckmann, Nirkhe 2022] which satisfy the Freedman-Hastings “NLTS conjecture”:



NLTS = $\exists e_c > 0$ such that *any* state $|\psi\rangle$ that satisfies $\langle\psi|H|\psi\rangle/N < e_c$ cannot be prepared via a constant depth circuit.

Can the Gibbs state of NLTS satisfying Hamiltonian be SRE?

Suggestive arguments that Gibbs state has no partition fn singularity at $T > 0$.



NLTS = $\exists e_c > 0$ such that *any* state $|\psi\rangle$ that satisfies $\langle\psi|H|\psi\rangle/N < e_c$ cannot be prepared via a constant depth circuit.

One can show that the Gibbs state of NLTS Hamiltonian in fact *cannot* be SRE for $T < T_c \neq 0$.

Basic idea: if it were SRE for all $T > 0$, i.e. if $e^{-\beta H}/Z \propto \sum_i p_i |\psi_i\rangle\langle\psi_i|$ where

$|\psi_i\rangle$ are all SRE, then the expectation value of energy density would exceed e_c , leading to a contradiction.

$\Rightarrow$ Separability transition in the Gibbs state without any partition fn singularity! (conjecture).

[Yu-Hsueh Chen, TG, 2310.07286; See also Hong, Guo, Lucas, 2403.10599: finite-T memory in these same Hamiltonians]

Summary and a few questions

- Separability criterion provides an organizing principle to classify mixed states as long range or short range entangled, with or without imposing symmetry.
- The decoding transition in several topological codes coincides with the separability transition: above the error threshold, the mixed state can be written as a convex sum of short-range entangled states.
- Other examples of separability transitions: mixed SPT states, spontaneous symmetry breaking, Gibbs state of NLTS Hamiltonians.
- How to distinguish distinct LRE mixed-states using separability?
- Relation to renormalization group (Sang, Zou, Hsieh, 2310.08639)?
- Field theoretic calculation of entanglement of proposed optimal pure states?
- Generalization to other topologically ordered/SPT states?
- Theory of separability transition in Gibbs state with no partition-fn singularity?