

Dissipation and Berry Phase Induced Transitions in Kondo lattices



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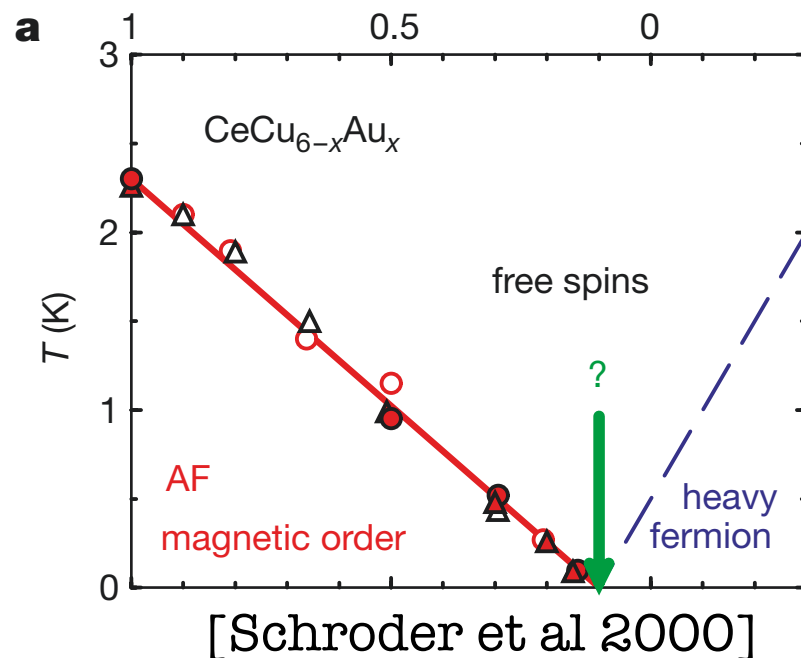
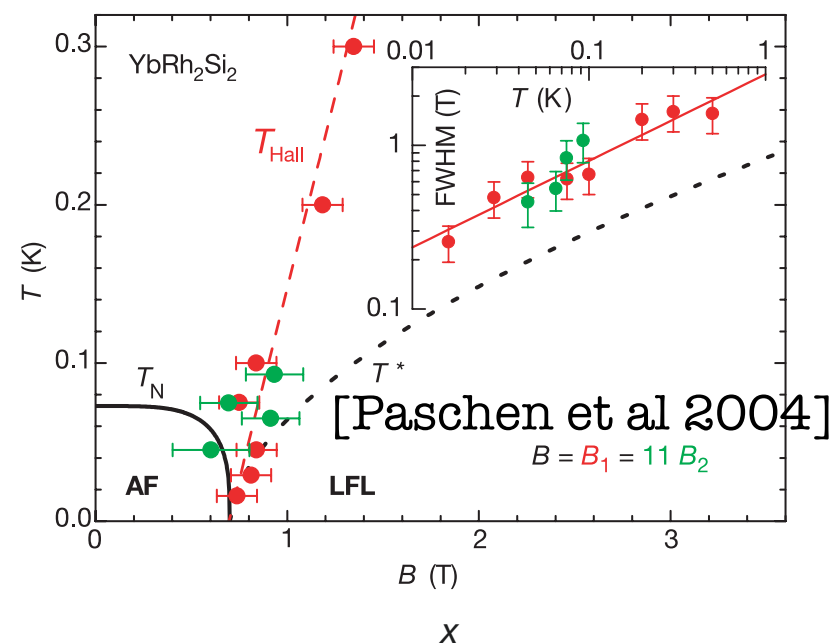


Matthias
Vojta
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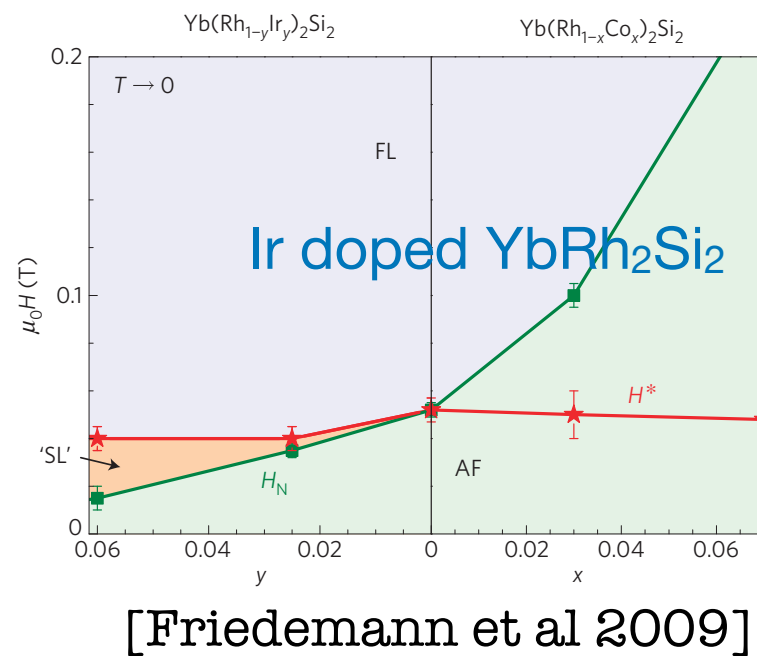
Main motivation for this talk:

Questions related to local moments in a metallic bath

Kondo breakdown transitions
(YbRh_2Si_2 , CeRhIn_5 , $\text{CeCu}_{6-x}\text{Au}_x$, ...)



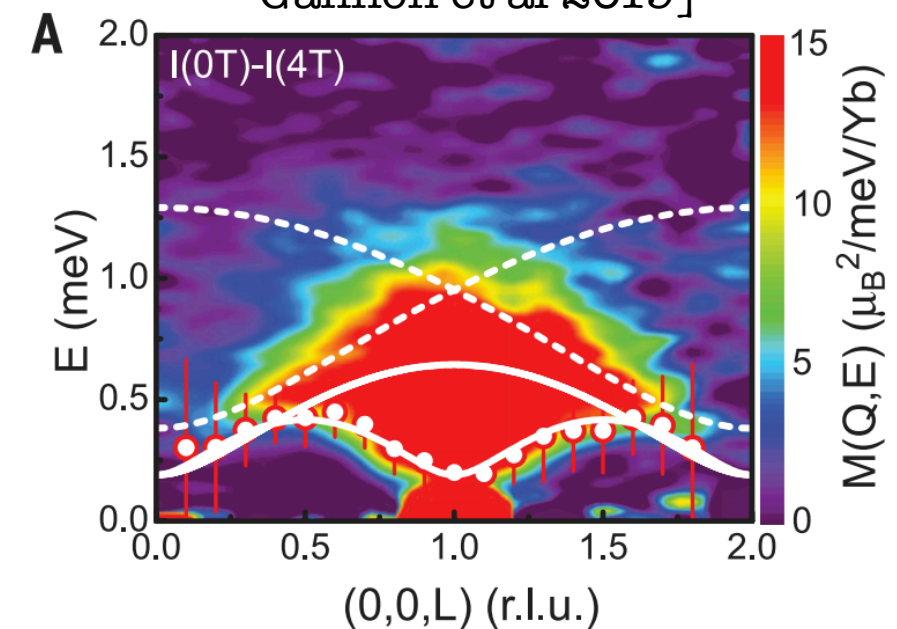
(Apparent) Non-Fermi liquid (stable) phases



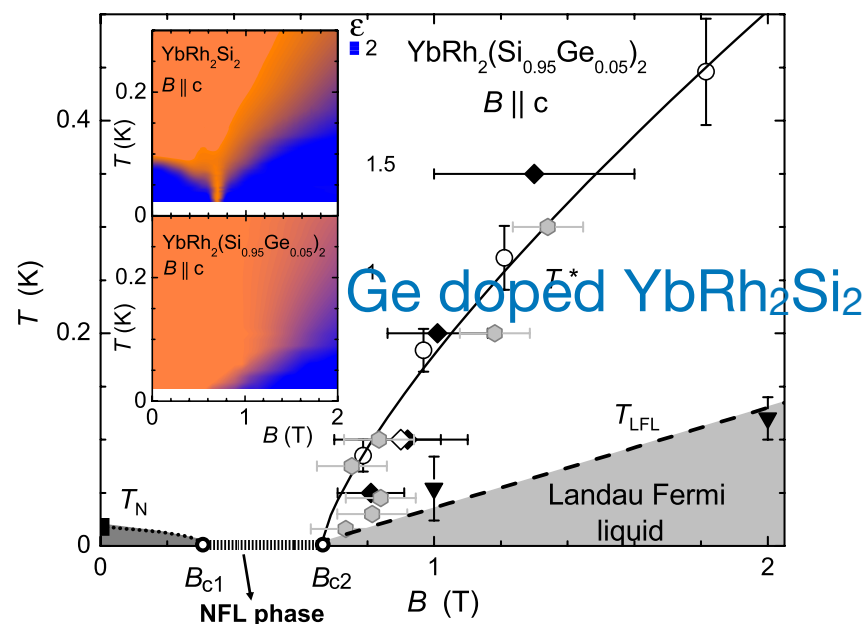
$\text{Yb}_2\text{Pt}_2\text{Pb}$

[Wu et al 2016;

Gannon et al 2019]



1d spinons in a 3D metal?

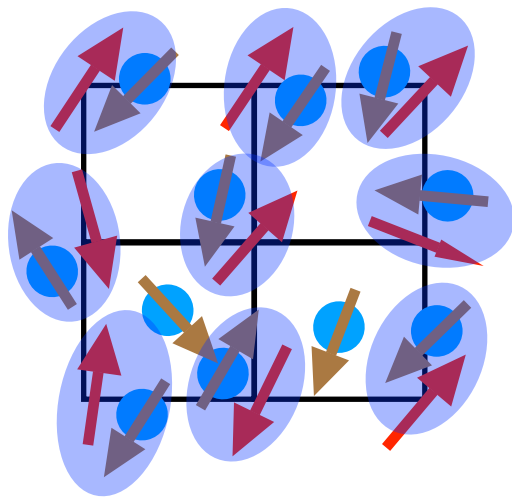


[Custers et al 2010]

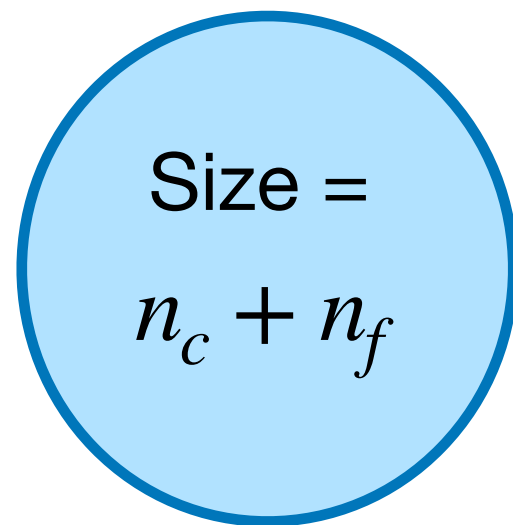
“Large” Vs “Small” Fermi surfaces

$$H = -t \sum_{\langle r, r' \rangle} \left(c_{r, \sigma}^\dagger c_{r', \sigma} + h.c. \right) + J_K \sum_r c_r^\dagger \sigma c_r \cdot \mathbf{S}_r + \sum_{r, r'} J_{r, r'} \mathbf{S}_r \cdot \mathbf{S}_{r'}$$

Heavy Fermi liquid
= Large Fermi surface

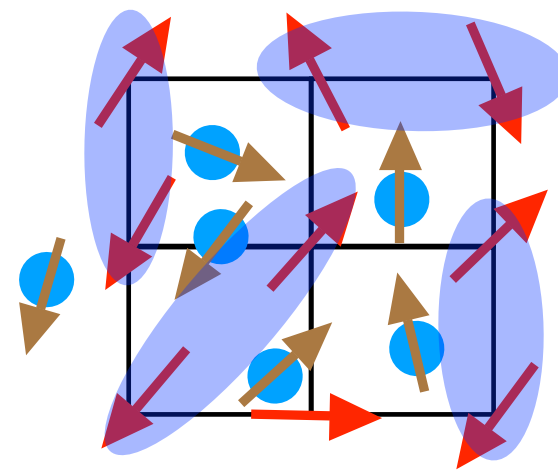


Real Space

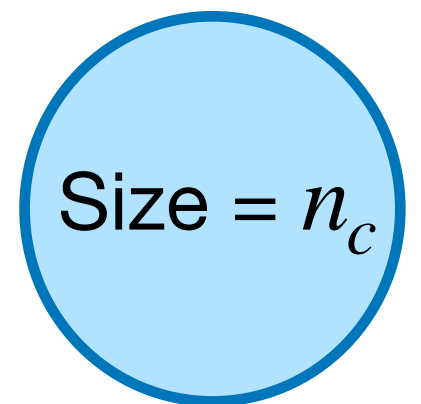


Momentum Space

“Fractionalized Fermi liquid”
= one route to small Fermi surface



Real Space



Momentum Space

A paramagnet with a small Fermi surface is necessarily a non-Fermi liquid [Senthil, Vojta, Sachdev 2002, Oshikawa 2000]. Further, magnetically ordered phases can also show Kondo breakdown [Si 2001, 2006].

1. At a generic chemical potential, Kondo lattice model has a fermion sign problem. Can one design models where one can access a paramagnetic large Fermi surface phase, and proximate phases, e.g., a Kondo-breakdown phase?

2. In low-dimensions (e.g. $d = 1$), Berry phase effects associated with order-parameter fluctuations can become important. Can they alter the phase diagram qualitatively, e.g., generate non-Fermi liquids?

Quantum critical phenomena

John A. Hertz

1976

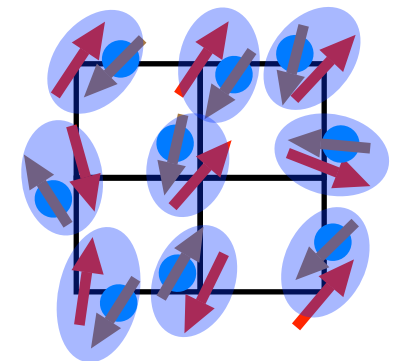
$$\Phi[\Psi] = \frac{1}{2} \sum_{\mathbf{q}\omega} \left(r_0 + q^2 + \frac{|\omega|}{q} \right) |\Psi(\mathbf{q}, \omega)|^2 + \text{interactions}$$

Sign Problem for Kondo Lattice models

$$H = -t \sum_{\langle r, r' \rangle} \left(c_{r, \sigma}^\dagger c_{r', \sigma} + h.c. \right) + J_K \sum_r c_r^\dagger \boldsymbol{\sigma} c_r \cdot \mathbf{S}_r + \sum_{r, r'} J_{r, r'} \mathbf{S}_r \cdot \mathbf{S}_{r'}$$

Absence of sign-problem requires half-filling and a bipartite lattice.

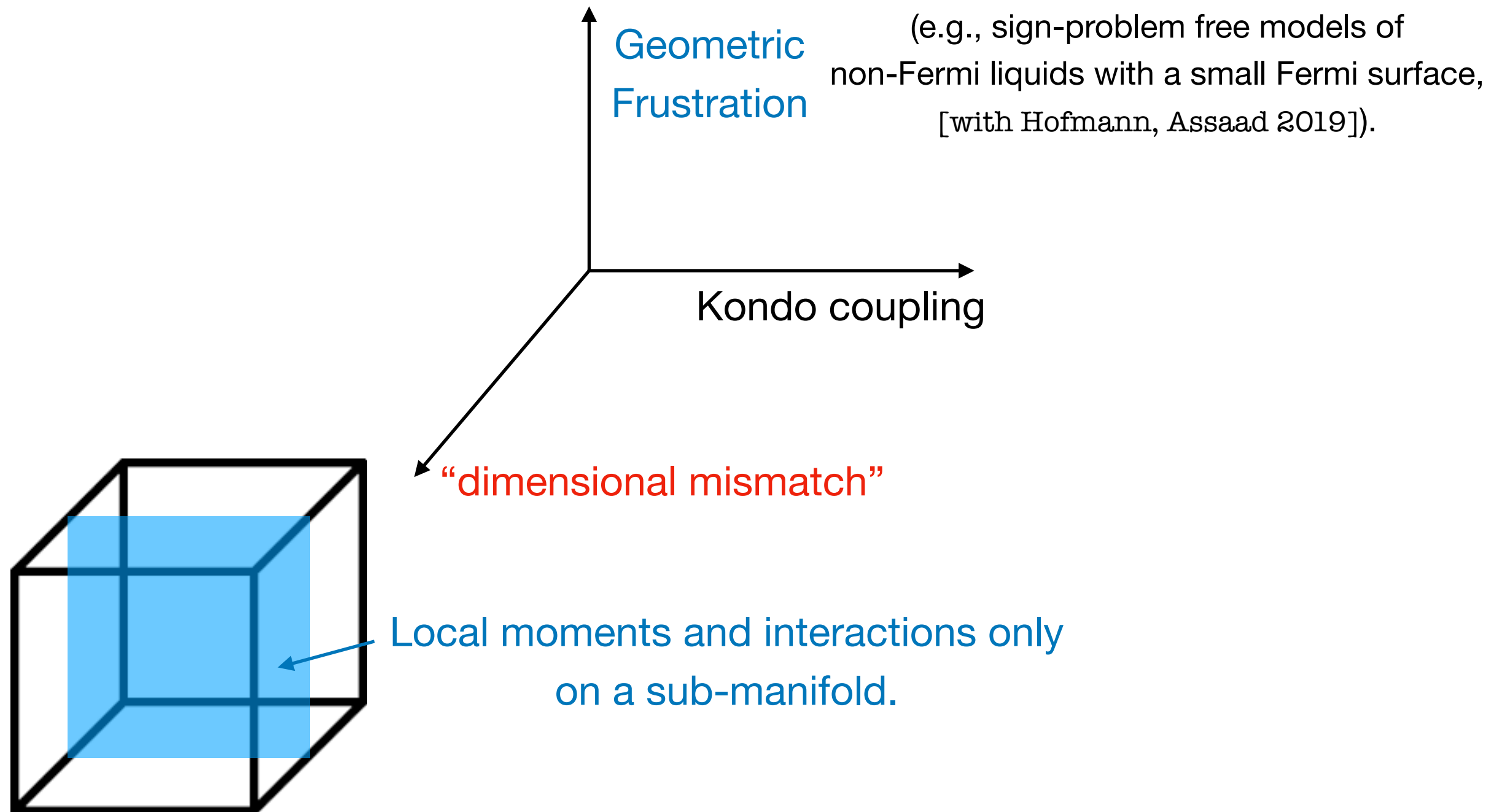
However, at half-filling, the heavy Fermi liquid is replaced by a
Kondo insulator $\Rightarrow$ No fermions at low energy.



More generally, tension between repulsively interacting fermions with
a stable Fermi surface, and sign-problem-free QMC

(see, e.g., [Ori Grossman, Erez Berg 2023]).

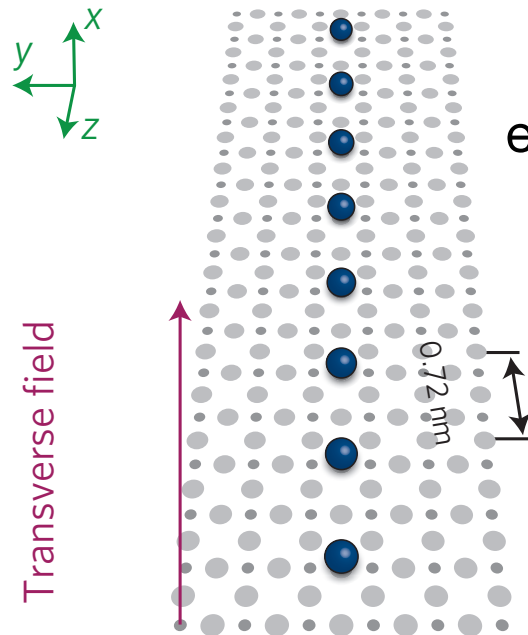
(At least) two ways to obtain a Kondo breakdown transition
without fermion sign problem...



Experiments on “Hybrid-dimensionality” Kondo lattice systems

Atomic spin-chain realization of a model for quantum criticality

R. Toskovic^{1†}, R. van den Berg^{2‡}, A. Spinelli¹, I. S. Eliens², B. van den Toorn¹, B. Bryant¹, J.-S. Caux²
and A. F. Otte^{1*} (2016)

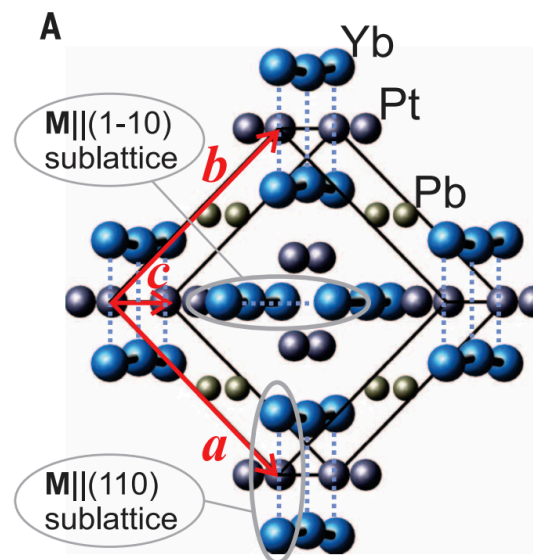


Magnetic cobalt adatoms on metallic copper.
effective spin-1/2 Co due to easy-axis anisotropy, Kondo coupled to Cu.

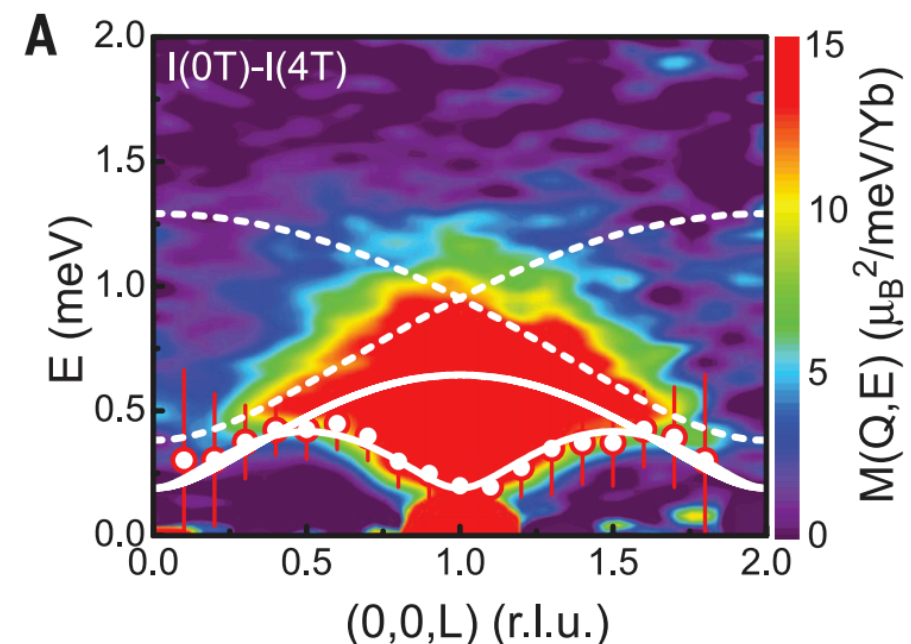
[Danu, Assaad, Mila (2019)]

Orbital-exchange and fractional quantum number excitations in an f-electron metal, Yb₂Pt₂Pb

L. S. Wu,^{1,2,3} W. J. Gannon,^{1,2,4} I. A. Zaliznyak,^{2*} A. M. Tsvetik,² M. Brockmann,^{5,6}
J.-S. Caux,⁶ M. S. Kim,² Y. Qiu,⁷ J. R. D. Copley,⁷ G. Ehlers,³
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[Wu et al 2016;
Gannon et al 2019]



Kondo breakdown in a hybrid-dimensionality model

1+1-D spin-chain Kondo coupled to a 2+1-D Dirac semi-metal.

Low energy
theory

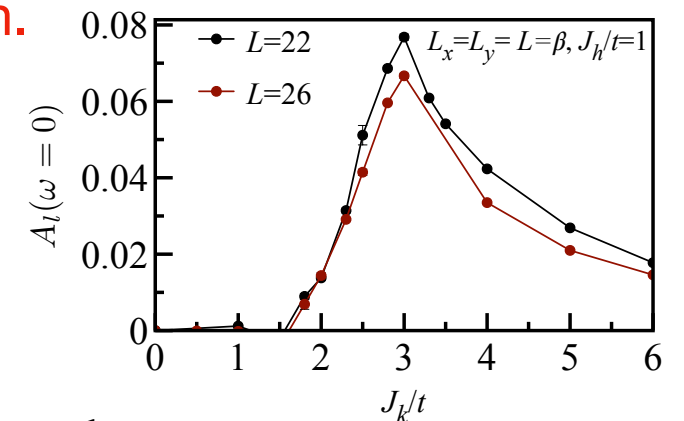
$$S = \int d^2x d\tau \bar{\Psi} \not{\partial} \Psi + J_K \int dx d\tau \vec{N} \cdot \bar{\Psi} \vec{\sigma} \Psi + S_{1d \text{ Heisenberg}}$$

[with Danu, Vojta, Assaad (2020)]

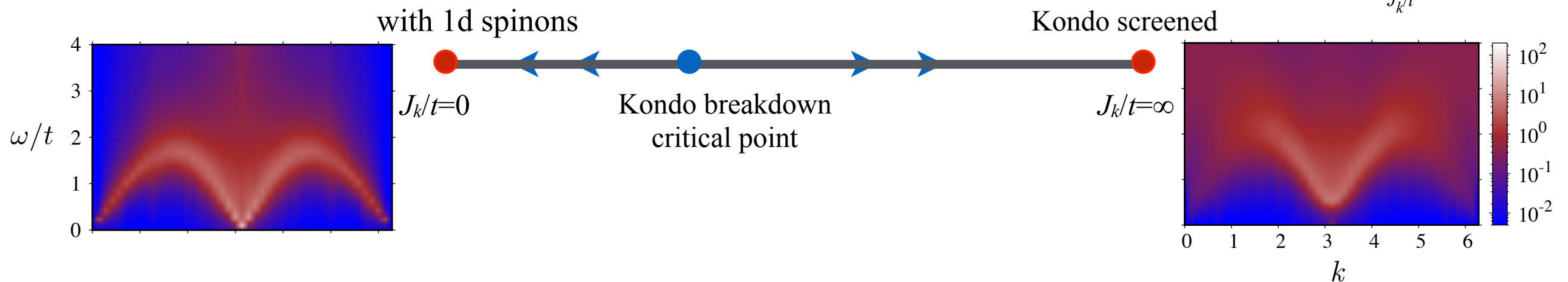
J_K irrelevant at the decoupled fixed-point corresponding to $|2+1\text{-D Dirac Fermions}\rangle \otimes |1+1\text{-D Heisenberg Chain}\rangle$.

Therefore, expect a Kondo breakdown transition.

$$\dot{j}_k = J_k \Lambda^\epsilon \quad \frac{dj_k}{d \ln \Lambda} = \epsilon j_k - \frac{j_k^2}{2} \quad (\epsilon = d - 3/2)$$



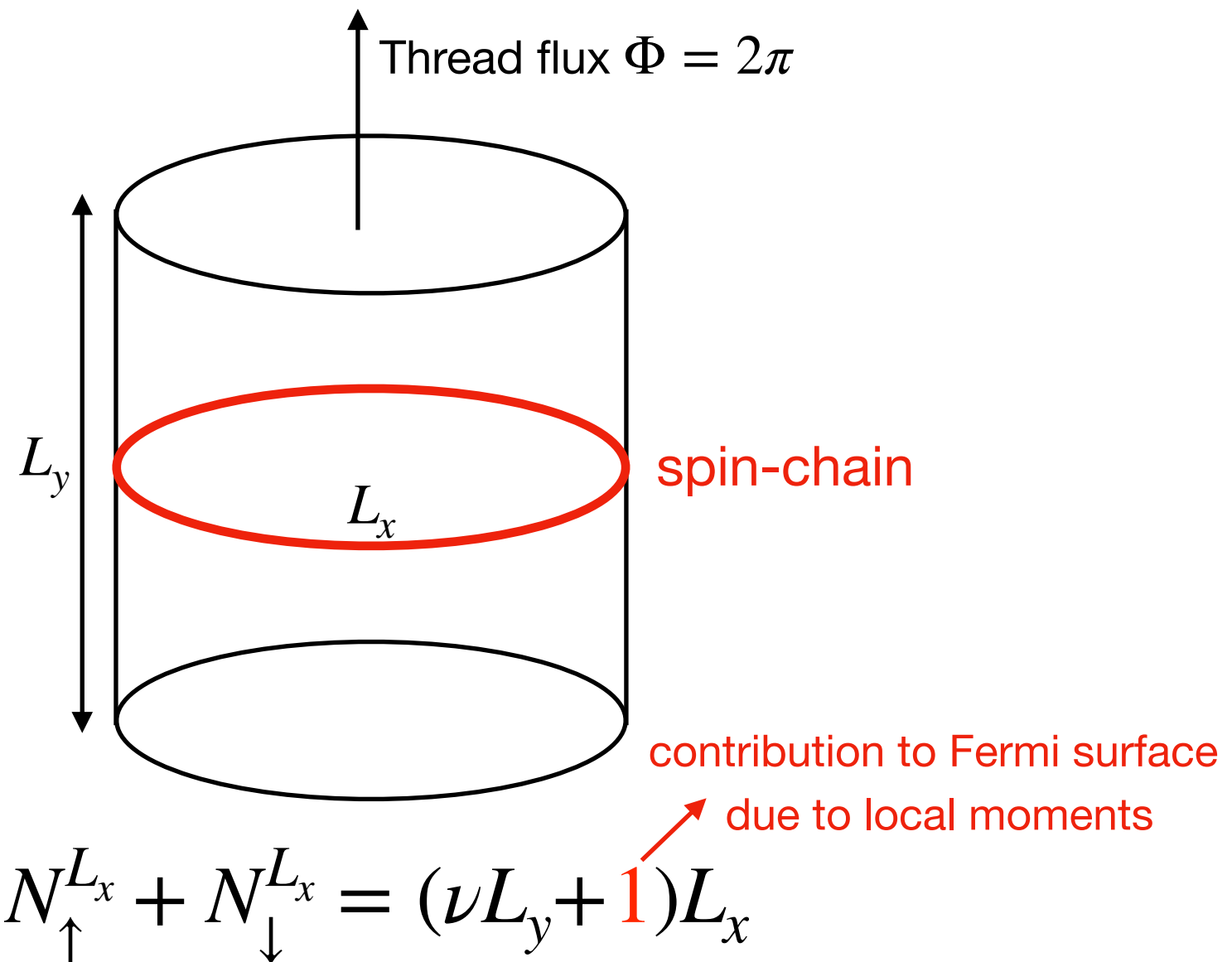
“Fractionalized Fermi liquid”



Spin-spin correlations decay as $(-1)^r \sqrt{\log(r)}/r$ for $J_K < J_{K,c}$ ($\sim$ Heisenberg chain)
and as $(-1)^r/r^4$ for $J_K > J_{K,c}$ ($\sim$ “large Fermi surface” semimetal)

Defining Large Fermi Surface in a hybrid-dimensionality system using Oshikawa's argument

[Oshikawa 2000]

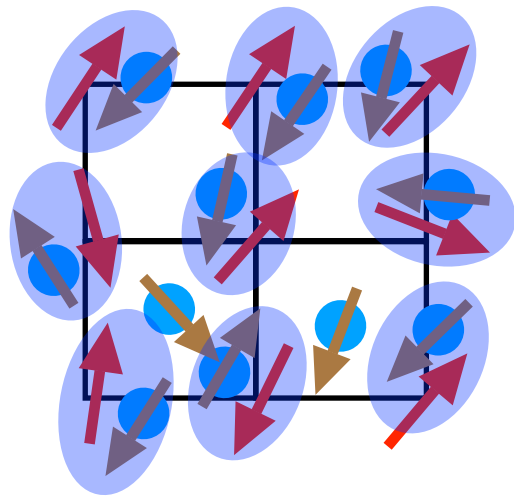


$N_{\uparrow/\downarrow}^{L_x}$ = Number of occupied momentum modes in the Fermi sea for the up/down electrons.

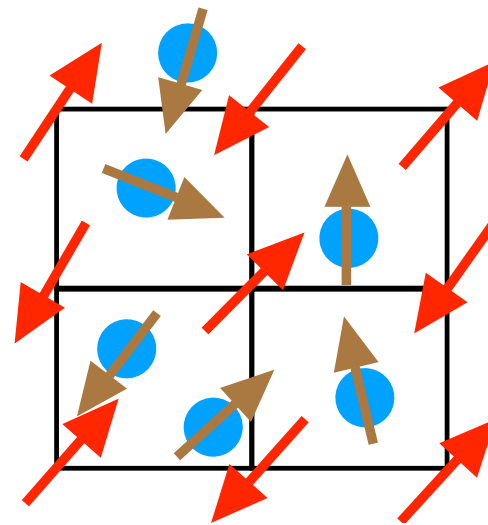
ν = Density of conduction electrons.

Transition between Magnetically Ordered Phase and Heavy Fermi liquid?

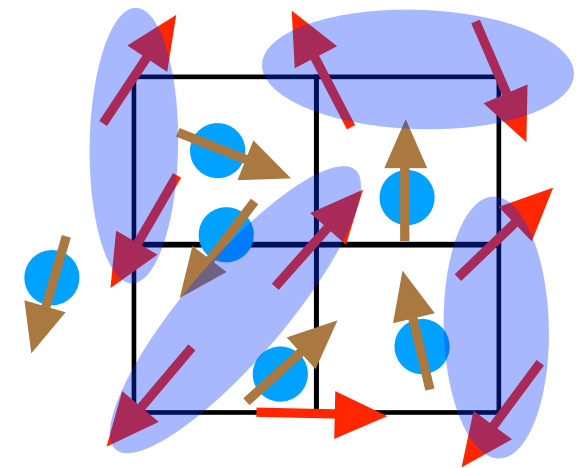
Heavy Fermi liquid



Magnetic metal



Fractionalized Fermi liquid



Again, fermion sign-problem a major obstacle.

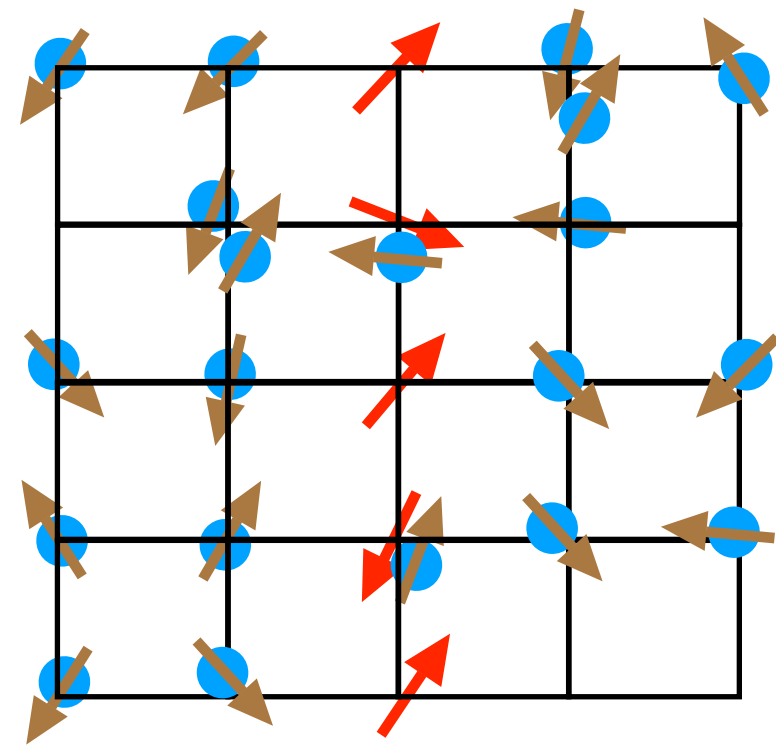
1d spin-chain on a 2d metal with Fermi surface

Square lattice nested
Fermi surface

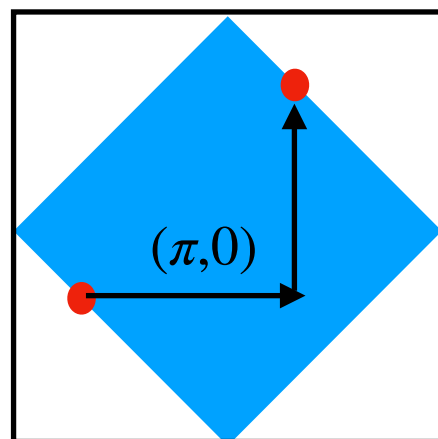
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \text{H.c.}) + \frac{J_k}{2} \sum_{l=1}^L \hat{c}_l^\dagger \boldsymbol{\sigma} \hat{c}_l \cdot \hat{S}_l$$

Kondo

$$+ J_h \sum_{l=1}^L \hat{S}_l \cdot \hat{S}_{l+\Delta l} \quad \text{1d Heisenberg}$$



Problem “intermediate” between the standard 2d Kondo lattice model,
and the standard single impurity Kondo problem



Scattering from spins couple Fermi surface points with $\Delta k_x \approx (\pi, 0)$

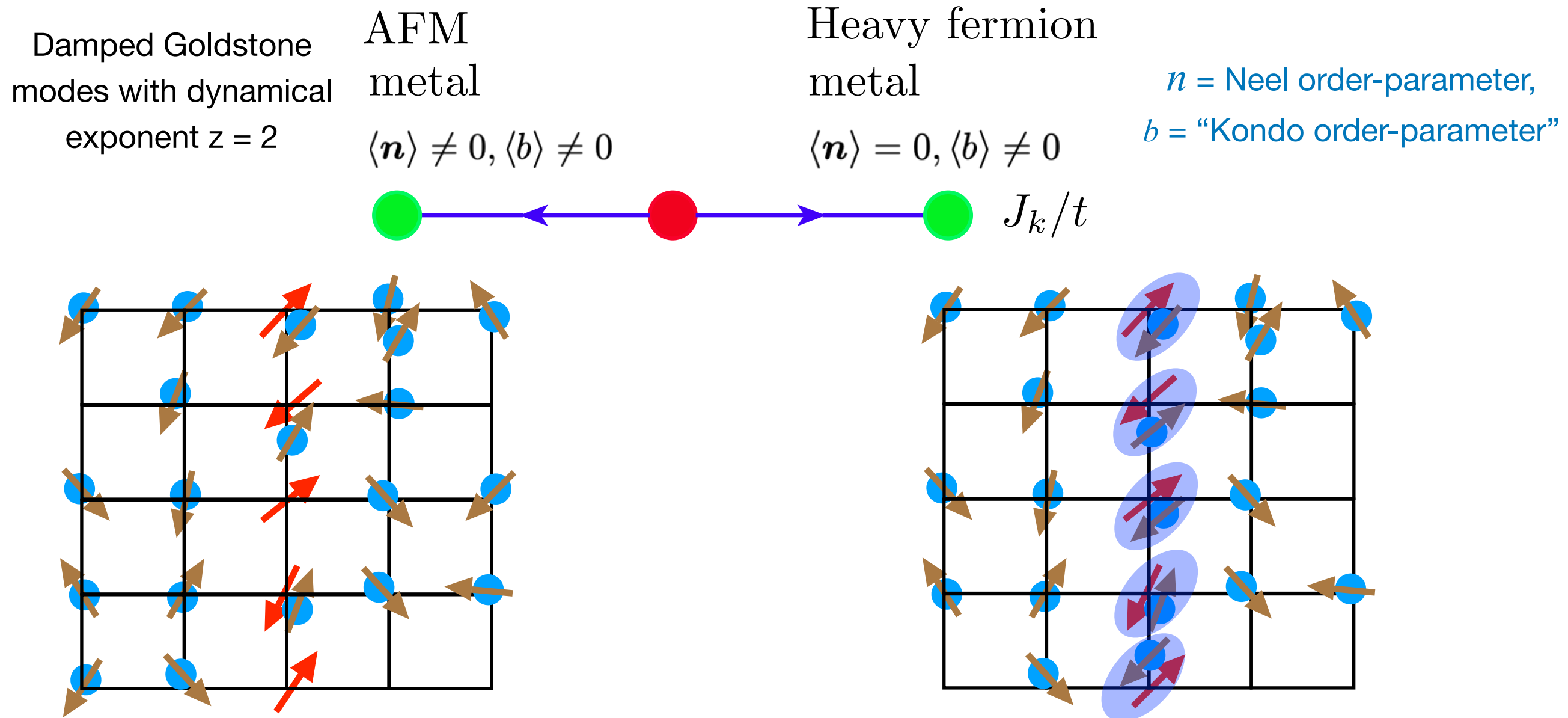
Fermi surface stable due to dimensional mismatch!

[with Danu, Vojta, Assaad 2022;
cf. Classen, Zaliznyak, Tsvelik 2018 → non-Fermi
liquid for a finite density of XXZ chains]

1d spin-chain on a 2d metal with Fermi surface

Dissipation due to fermions can lead to long-range AFM order even in a 1d chain!

[e.g. P. Werner, M. Troyer, and S. Sachdev 2005;
Lobos, Cazalilla, Chudzinski 2012; Weber, Luitz, Assaad 2021]



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Damped Goldstone
modes with dynamical
exponent $z = 2$

AFM
metal

$$\langle \mathbf{n} \rangle \neq 0, \langle \mathbf{b} \rangle \neq 0$$

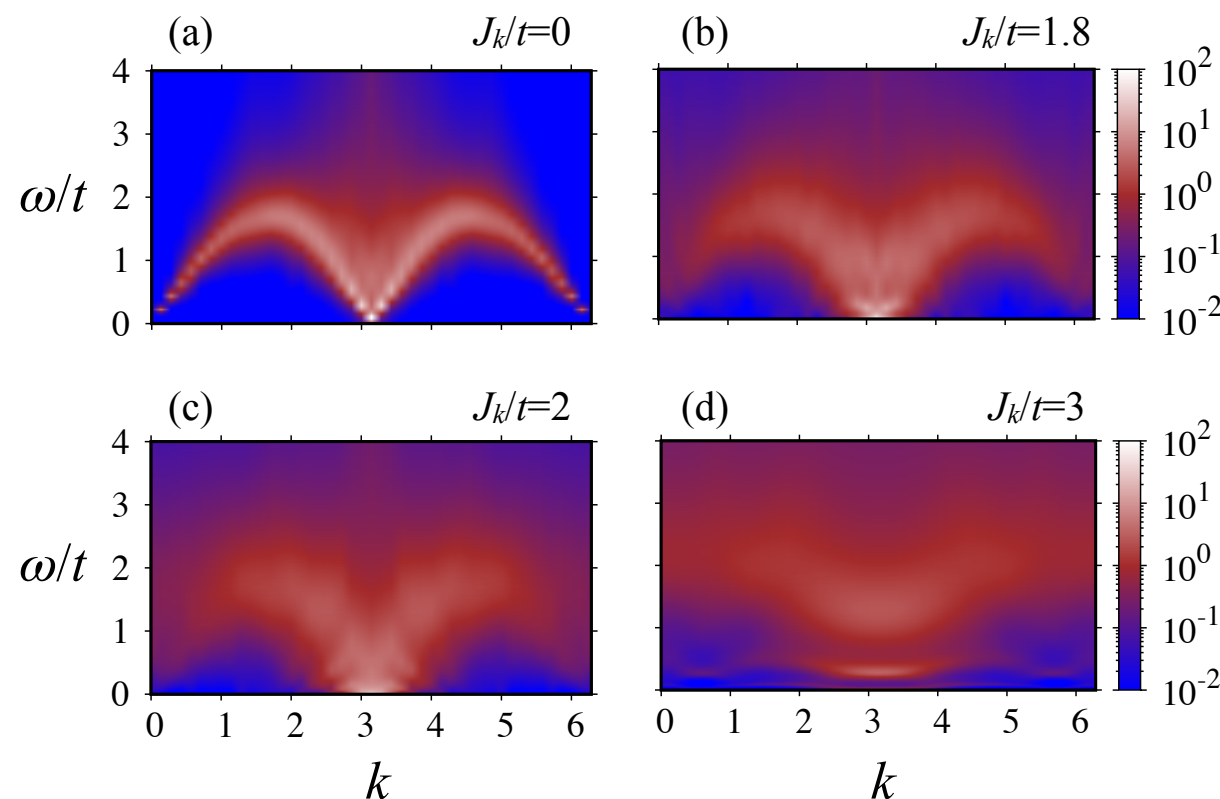
Heavy fermion
metal

$$\langle \mathbf{n} \rangle = 0, \langle \mathbf{b} \rangle \neq 0$$

$\mathbf{n}$ = Neel order-parameter,
 $\mathbf{b}$ = “Kondo order-parameter”

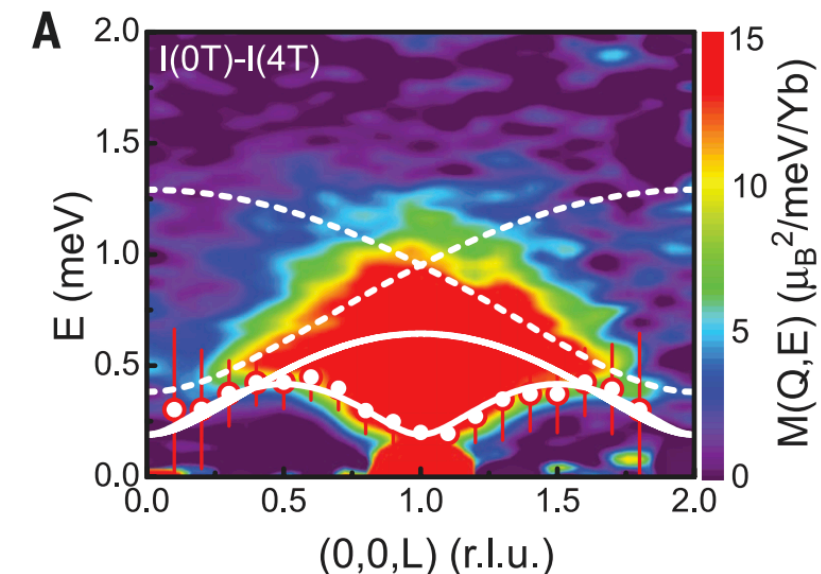


$$S(\mathbf{k}, \omega) = \frac{\text{Im} \chi(\mathbf{k}, \omega + i0^+)}{1 - e^{-\beta \omega}}$$



**Orbital-exchange and fractional
quantum number excitations in
an f-electron metal, Yb₂Pt₂Pb**

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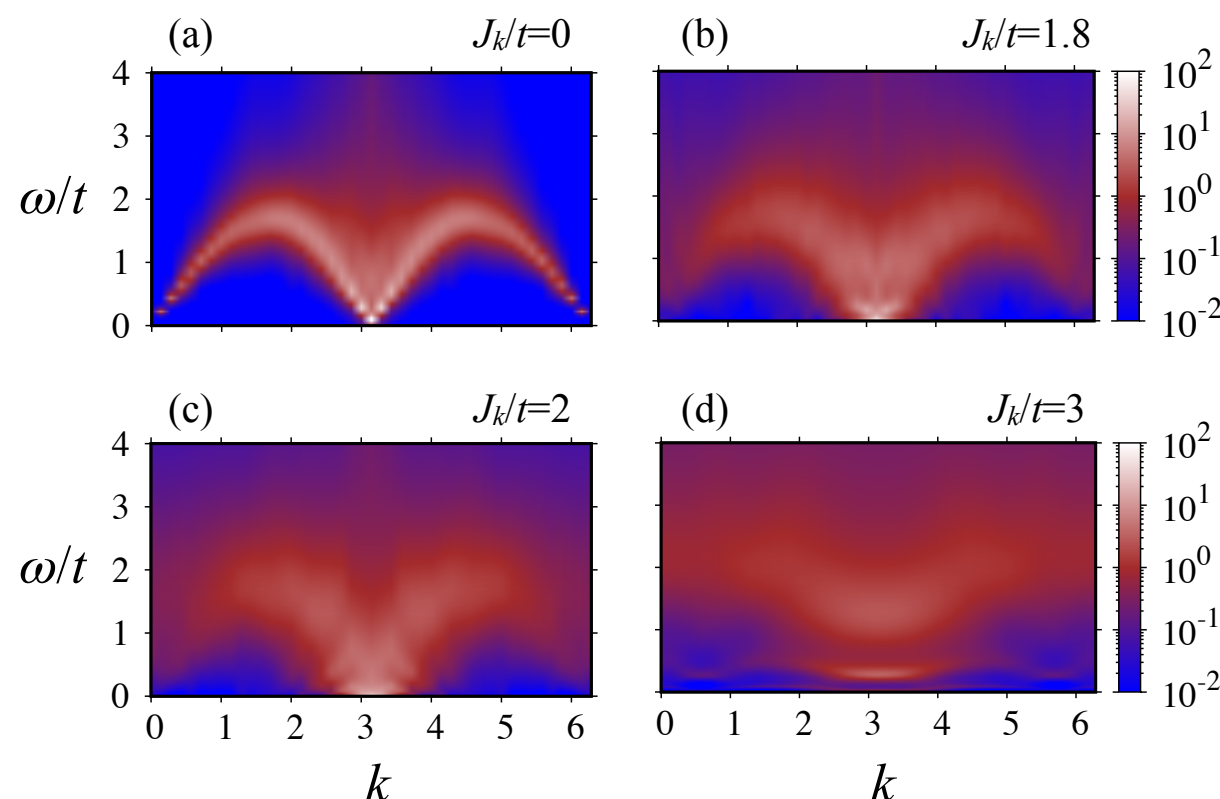
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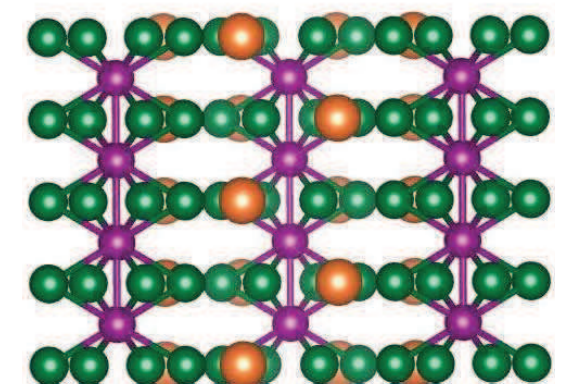


$$S(\mathbf{k}, \omega) = \frac{\text{Im} \chi(\mathbf{k}, \omega + i0^+)}{1 - e^{-\beta \omega}}$$



Ti₄MnBi₂: another metallic system with
local moments along chains

[Pandey...Aronson et al 2000]



$$\langle \text{Neel} \rangle \approx 0.05 \mu_B$$

Stability of Dissipation induced O(3) symmetry breaking

$$\mathcal{S}_{\text{diss}}(\mathbf{n}) = \frac{\Gamma}{2} \int d\tau d\tau' dr \frac{\mathbf{n}(r, \tau) \cdot \mathbf{n}(r, \tau')}{(\tau - \tau')^2} + \frac{\rho_s}{2} \int d\tau dr (\partial_\tau \mathbf{n}(r, \tau))^2 + (\partial_r \mathbf{n}(r, \tau))^2$$

$$\mathbf{n}(r, \tau) = \left(\boldsymbol{\sigma}(r, \tau), \sqrt{1 - \boldsymbol{\sigma}(r, \tau)^2} \right) \quad \mathcal{S}_{\text{diss}}(\boldsymbol{\sigma}) = \mathcal{S}_0(\boldsymbol{\sigma}) + \mathcal{S}_1(\boldsymbol{\sigma}) + \dots$$

$$\mathcal{S}_0(\boldsymbol{\sigma}) = \frac{\Gamma}{2} \int d\tau d\tau' dr \frac{\boldsymbol{\sigma}(r, \tau) \cdot \boldsymbol{\sigma}(r, \tau')}{(\tau - \tau')^2} + \frac{\rho_s}{2} \int d\tau dr (\partial_r \boldsymbol{\sigma}(r, \tau))^2,$$

Invariant under $r \rightarrow \lambda r, \tau \rightarrow \lambda^2 \tau, \sigma(r, \tau) \rightarrow \sigma(r, \tau) / \sqrt{\lambda}$.

Interaction between damped
Goldstone modes

$$\mathcal{S}_1(\boldsymbol{\sigma}) = \frac{\Gamma}{8} \int d\tau d\tau' dr \frac{\boldsymbol{\sigma}^2(r, \tau) \cdot \boldsymbol{\sigma}^2(r, \tau')}{(\tau - \tau')^2} + \frac{\rho_s}{2} \int d\tau dr (\boldsymbol{\sigma}(r, \tau) \cdot \partial_r \boldsymbol{\sigma}(r, \tau))^2$$

$\mathcal{S}_1(\boldsymbol{\sigma}) \rightarrow \mathcal{S}_1(\boldsymbol{\sigma}) / \lambda \Rightarrow$ Interactions between Goldstones RG irrelevant

Field theory for d=1 Spin Chain coupled to d > 1 Free fermions?

Weak-coupling approach: $SU(N)_k$ WZW CFT perturbed by dissipation
(= Hertz-Millis + Berry Phase)

(Kinetic energy) $S_{\text{Grad}} = \frac{1}{\lambda} \int d\tau dx \text{tr} \left(\frac{1}{c^2} \partial_\tau g \partial_\tau g^{-1} + \partial_x g \partial_x g^{-1} \right) \quad (g = SU(N) \text{ matrix})$

(Berry phase) $S_{\text{WZ}} = -\frac{i k}{12\pi} \int d^3 y \epsilon^{ijk} \text{Tr}[g^{-1} \partial_i g g^{-1} \partial_j g g^{-1} \partial_k g]$

(Landau damping) $S_{\text{Dis}} = k^2 \gamma \int d\tau d\tau' dx K(\tau - \tau') \text{tr}[\mathbb{1} - g(\tau, x) g^{-1}(\tau', x)]$

$$K(\tau - \tau') = \frac{A}{|\tau - \tau'|^{3-\delta}} \quad \delta = \tilde{\delta}/k, \quad \tilde{\delta} = O(1)$$



[work with Simon Martin
(to appear soon)]

Fixed point annihilation for a spin in a fluctuating field

Adam Nahum¹ (2022)

$$\mathcal{S} = \frac{1}{2g} \int dt dt' K(t - t') (\vec{n}(t) - \vec{n}(t'))^2 - iS \Omega[\vec{n}]. \quad \text{“Bose-Kondo model”}$$

See also [Cuomo et al 2022; Hu, Si 2022; Weber, Vojta 2023; Cai, Si 2019]

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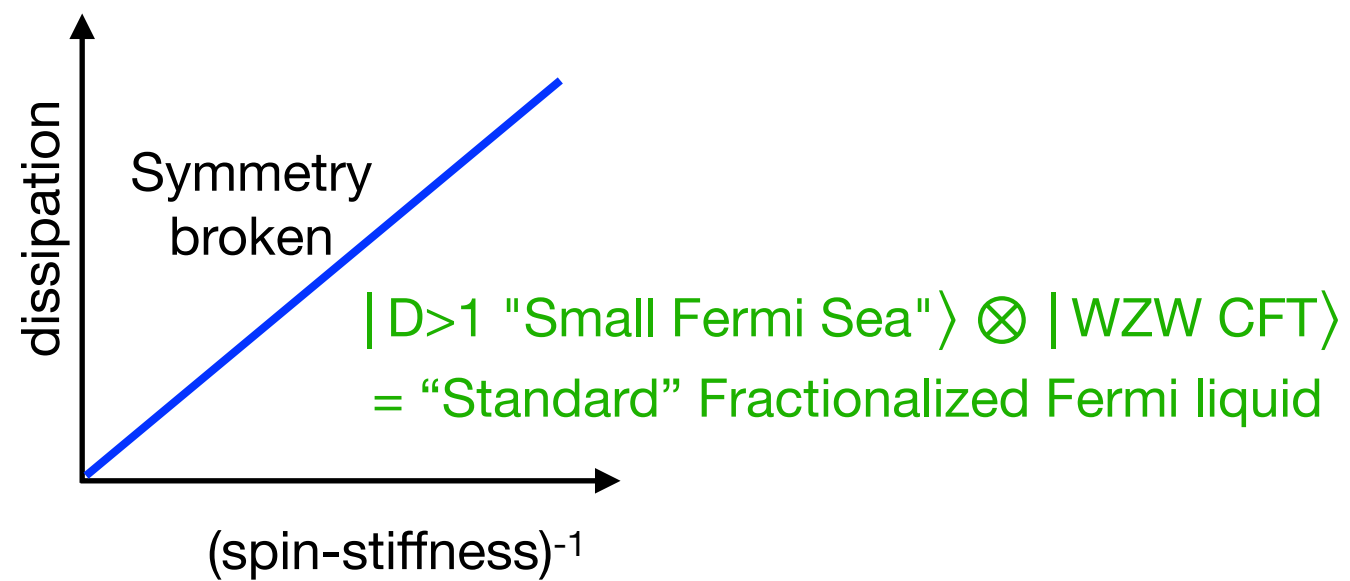
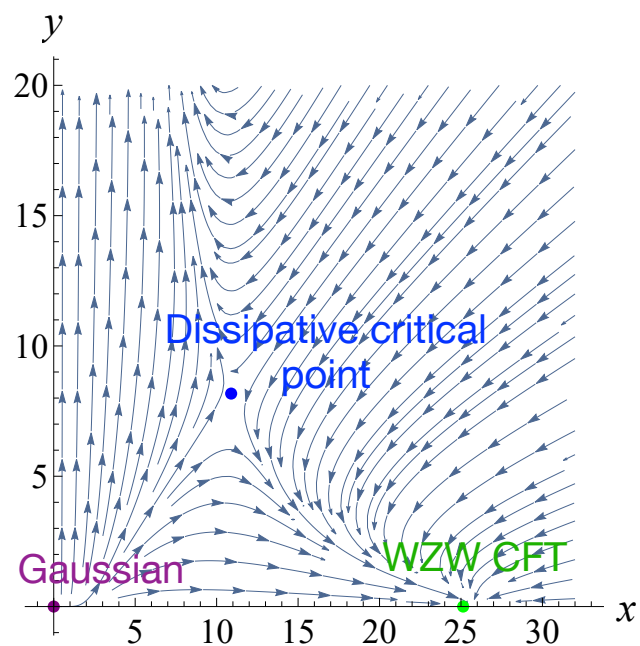
$$K(\tau - \tau') = \frac{A}{|\tau - \tau'|^{3-\delta}} \quad \delta = \tilde{\delta}/k, \quad \tilde{\delta} = O(1)$$



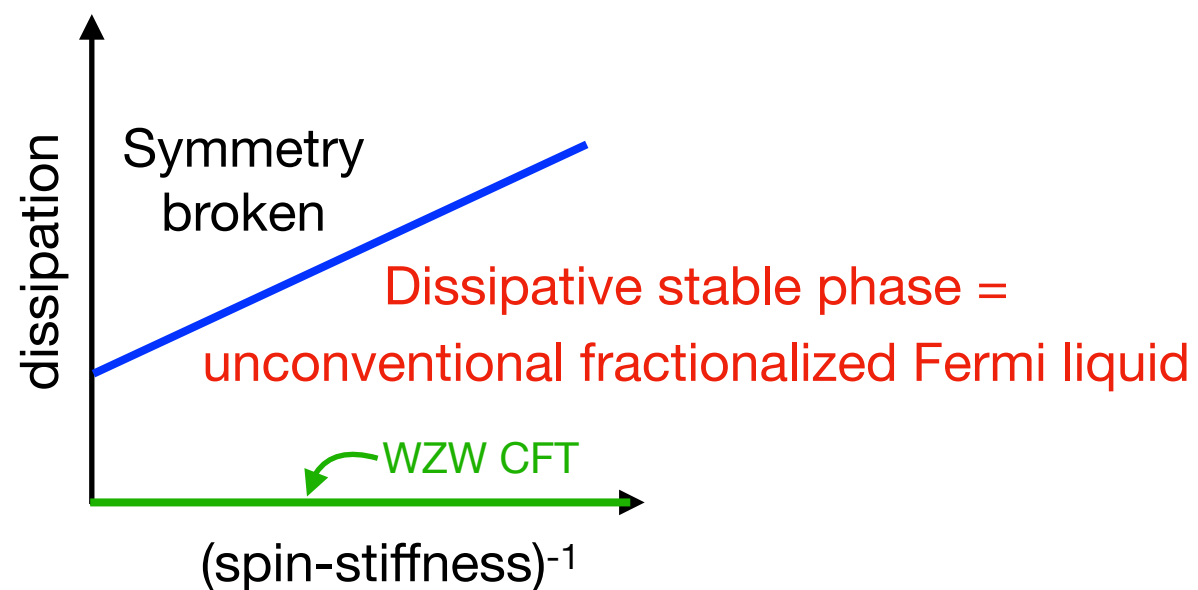
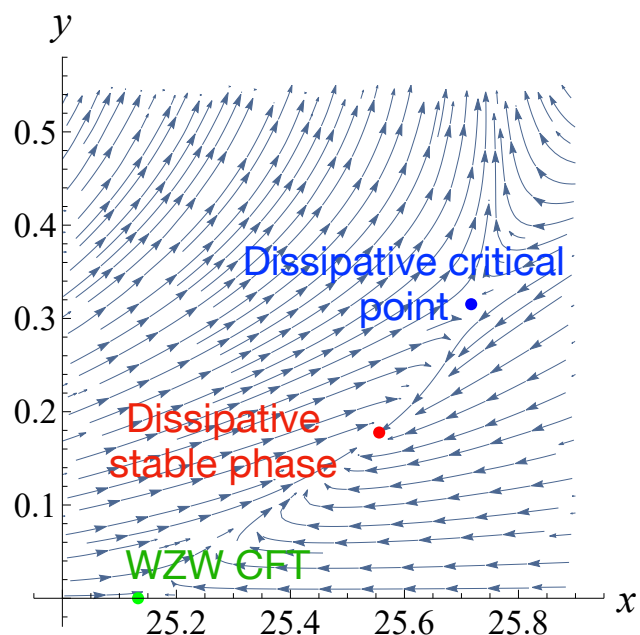
[work with Simon Martin
(to appear soon)]

Spin-1/2 Heisenberg chain with ohmic dissipation corresponds to $N = 2, k = 1, \delta = 1$. In this case, dissipation marginally relevant, and numerics indicates it induces ordering [Lafroncie, Affleck, Berciu 2005; Weber, Luitz, Assaad 2021].

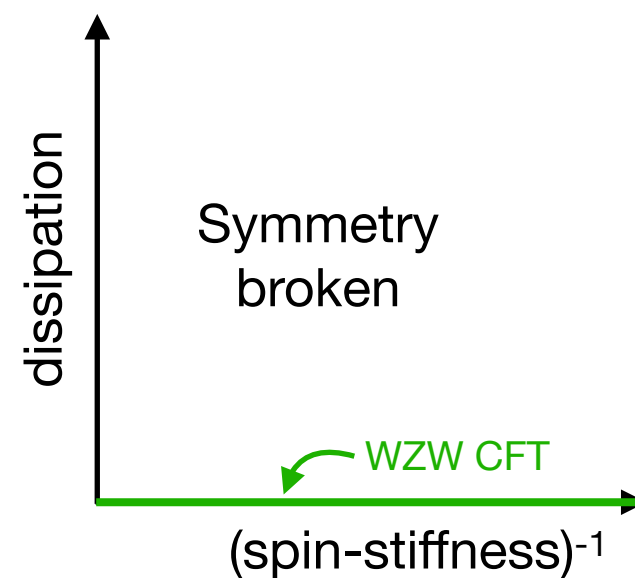
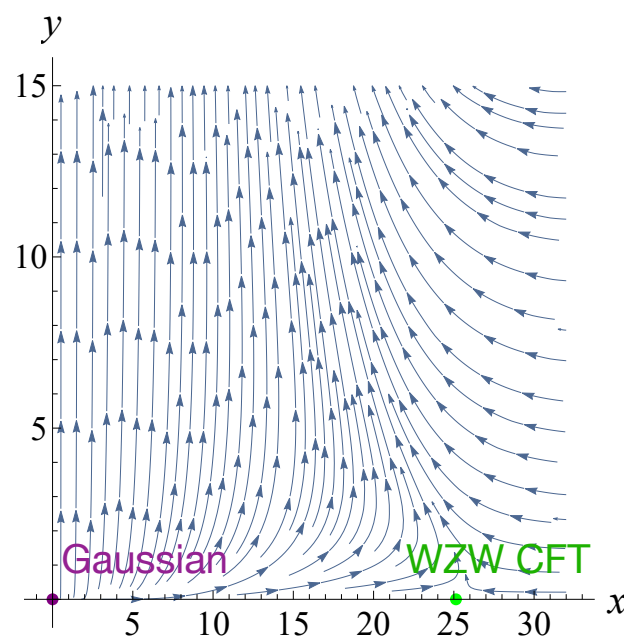
$$\tilde{\delta} < \tilde{\delta}_1$$

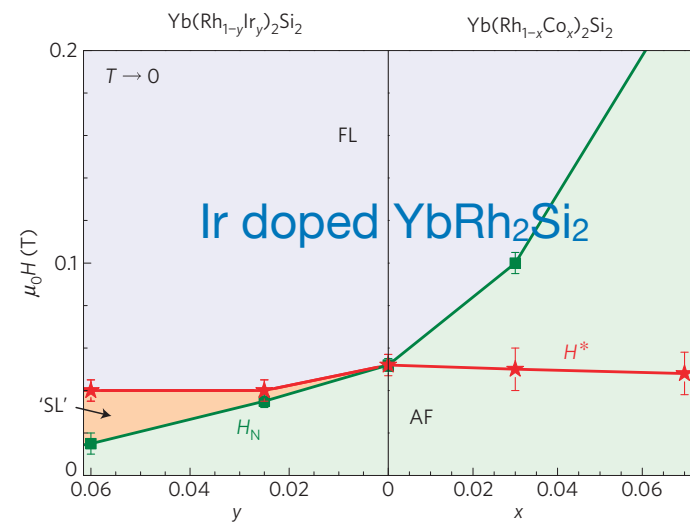


$$\tilde{\delta}_1 < \tilde{\delta} < \tilde{\delta}_2$$

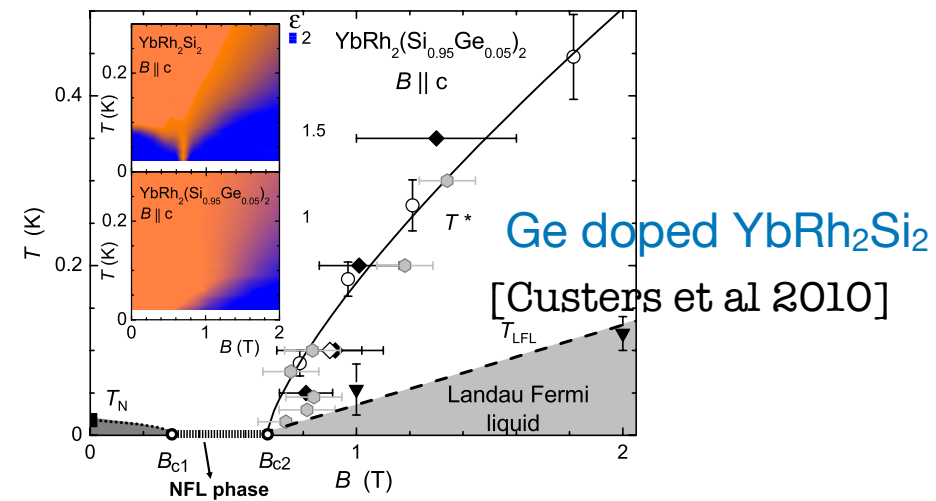


$$\tilde{\delta} > \tilde{\delta}_2$$

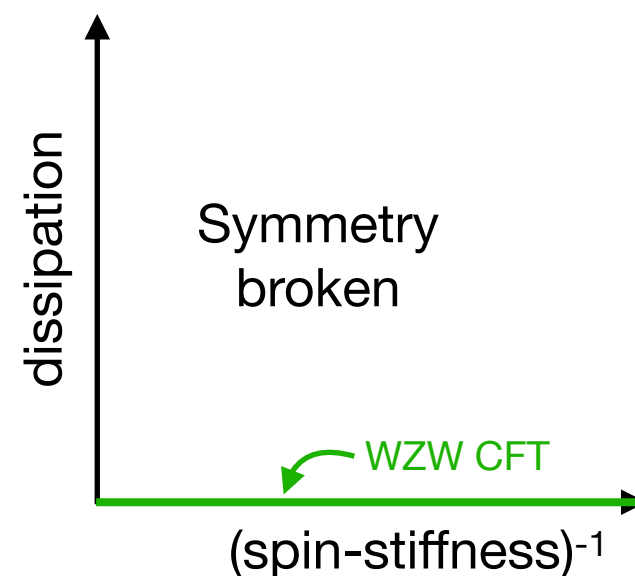
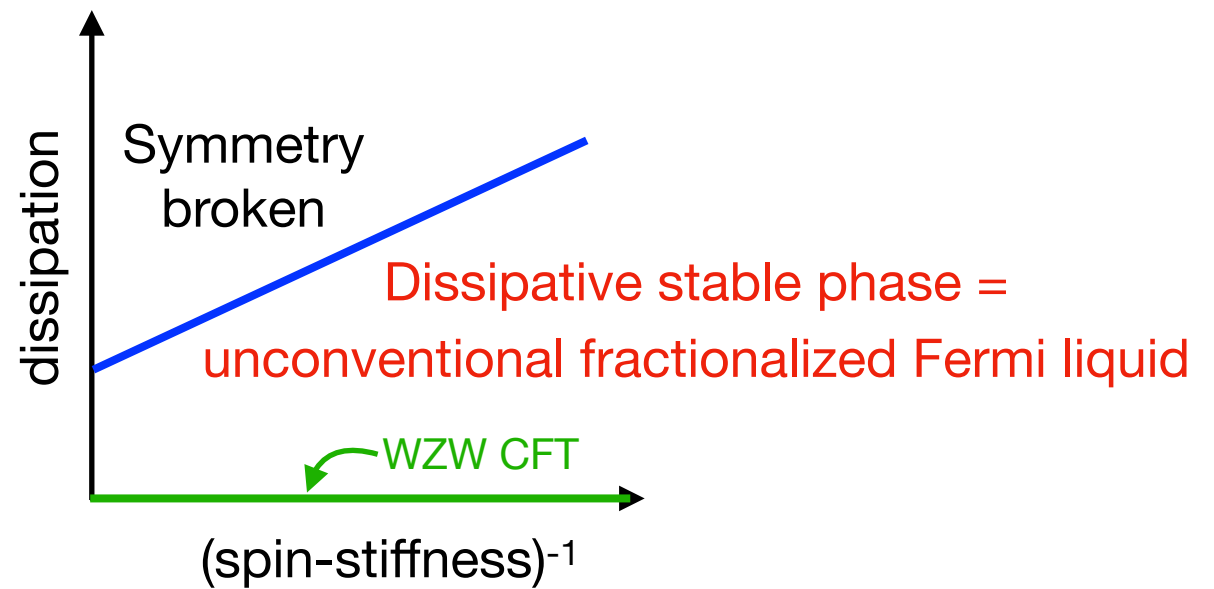
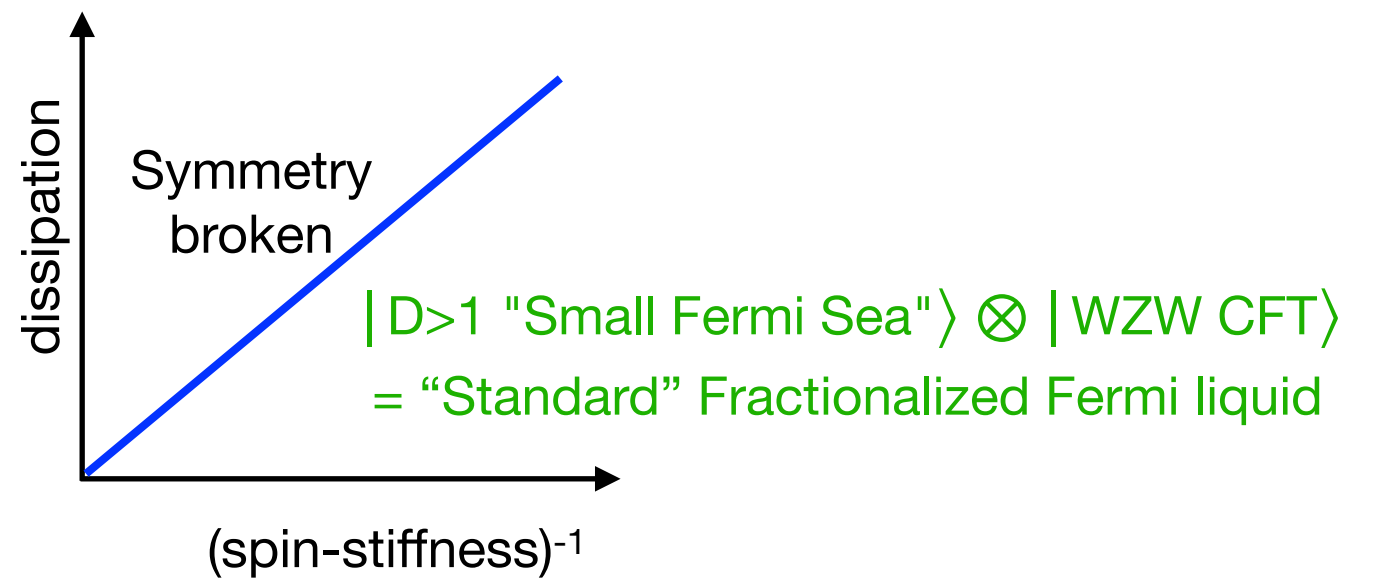
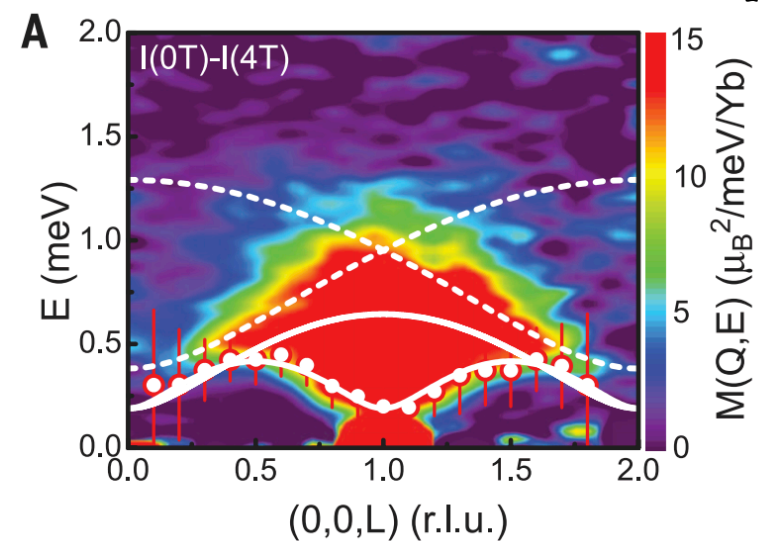




[Friedemann et al 2009]



Yb₂Pt₂Pb [Wu et al 2016; Gannon et al 2019]



$\pi/2$ space-time rotation of dissipative spin-chain =
Hamiltonians of spatially long-range interacting spin-chains

$$\frac{\vec{S}(x, \tau) \cdot \vec{S}(x, \tau')}{(\tau - \tau')^\alpha} \Rightarrow \frac{\vec{S}(x, \tau) \cdot \vec{S}(x', \tau)}{(x - x')^\alpha}$$

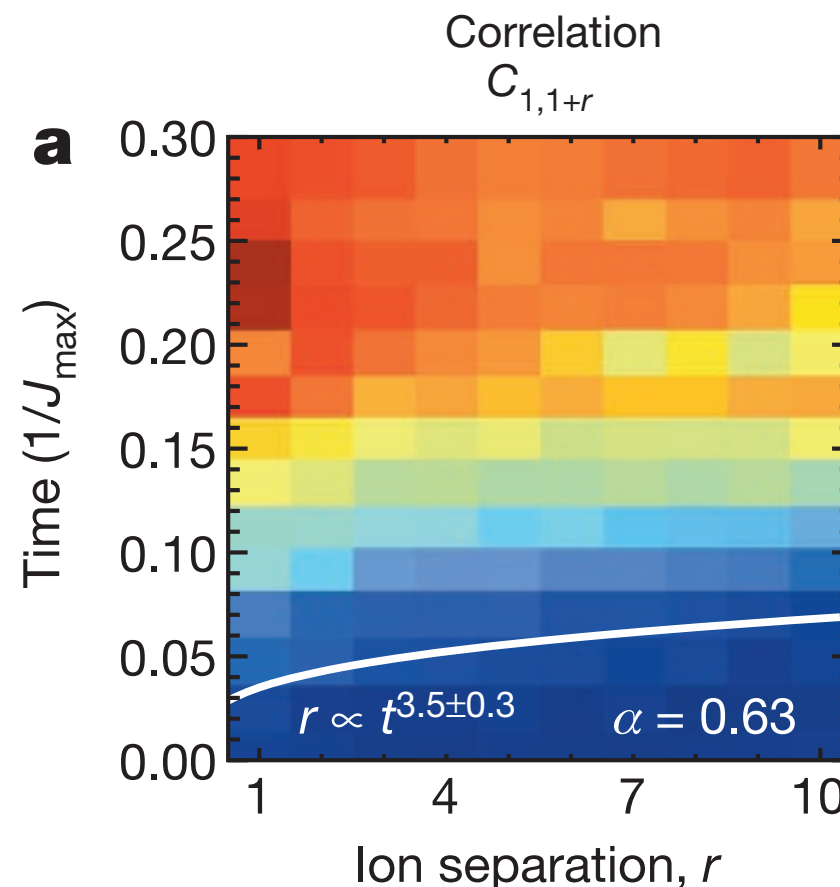
Several platforms: ion traps, cavity mediated interactions, dipolar systems, Rydberg atoms...

$$H_{XY} = \frac{1}{2} \sum_{i < j} J_{i,j} (\sigma_i^x \sigma_j^x + \sigma_i^z \sigma_j^z)$$

$$J_{i,j} \propto 1/|i - j|^\alpha$$

[Richerme, ..., Monroe 2014]

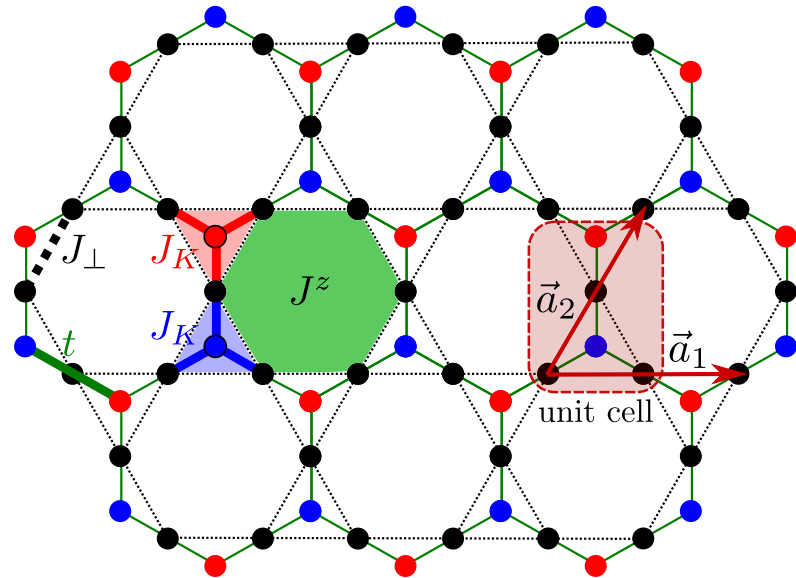
Typically, $0 < \alpha < 3$.



Summary and some questions

- Broad message: possible to construct sign-problem-free models that sometime allow unbiased simulation of physics potentially relevant to Heavy fermions, e.g., non-Fermi liquids, Heavy fermions, Landau damping induced order, ...
- New, stable critical phases in dissipative $SU(N)_k$ spin-chains. Interesting RG flow with fixed-point annihilation reminiscent of Bose-Kondo single-impurity problem. Potentially new, infinite class of CFTs in a relativistic analog.
- Nature of quantum critical point in hybrid-dimensionality systems with Fermi surface using Polchinski/Shankar RG? Detailed understanding of Yb_2Pt_2Pb ? Relation to c-theorems for defect CFTs, and logarithmic BCFTs? ([Metlitski 2020; Cuomo, Zhang 2023]).

Kondo breakdown via geometric frustration



- ● c fermions (honeycomb lattice)
- localized spins (kagome lattice)

$$\hat{H} = \hat{H}_c + \hat{H}_S + \hat{H}_K$$

$$\hat{H}_c = -t \sum_{\langle \mathbf{x}, \mathbf{y} \rangle, \sigma} \hat{c}_{\mathbf{x}, \sigma}^\dagger \hat{c}_{\mathbf{y}, \sigma} + h.c.$$

$$\hat{H}_S = -J^\perp \sum_{\langle \mathbf{i}, \mathbf{j} \rangle} \left(\hat{S}_{\mathbf{i}}^{f,+} \hat{S}_{\mathbf{j}}^{f,-} + h.c. \right) + J^z \sum_{\hexagon} \left(\hat{S}_{\hexagon}^{f,z} \right)^2$$

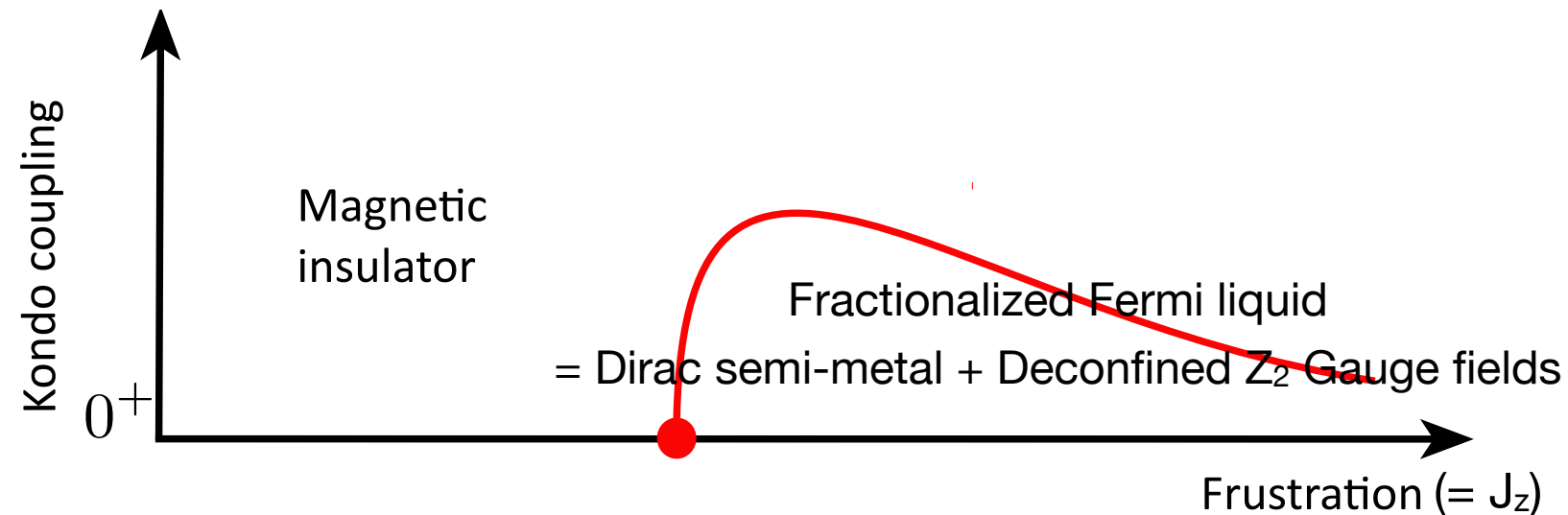
$$\hat{H}_K = J_K \sum_{\langle \mathbf{x}, \mathbf{i} \rangle} \left[\hat{S}_{\mathbf{x}}^{c,z} \hat{S}_{\mathbf{i}}^{f,z} - (-1)^{\mathbf{x}} \left(\hat{S}_{\mathbf{x}}^{c,+} \hat{S}_{\mathbf{i}}^{f,-} + h.c. \right) \right]$$

[Hofmann, Assaad, TG 2019]

H_s is the “Balents-Fisher-Girvin” model that supports a topologically ordered Z_2 spin-liquid when $J^z \gg J^\perp$

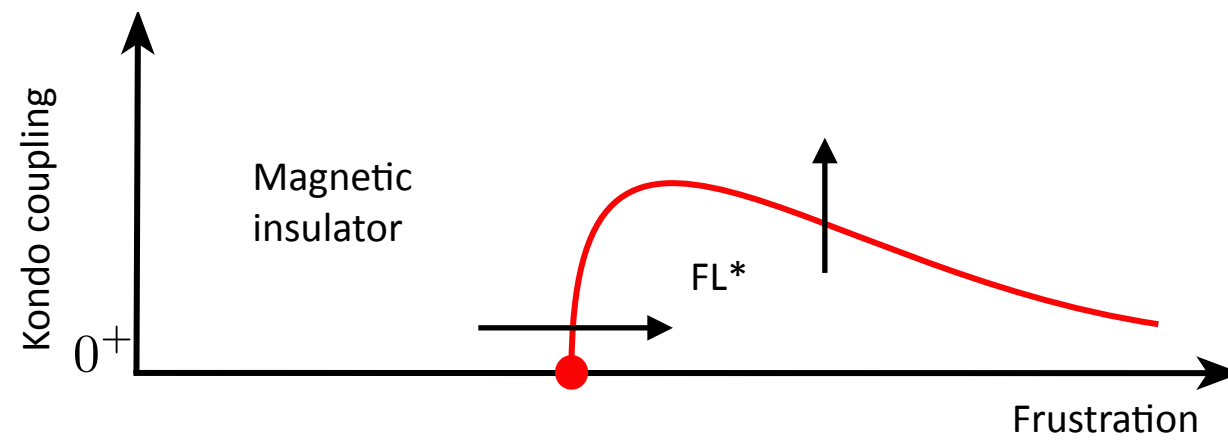
The above model does not have a sign-problem at half-filling,
(via employing fermion representation of spins).

Schematic Phase Diagram obtained from QMC



FL* phase has a small Fermi surface, i.e. violates Luttinger theorem.

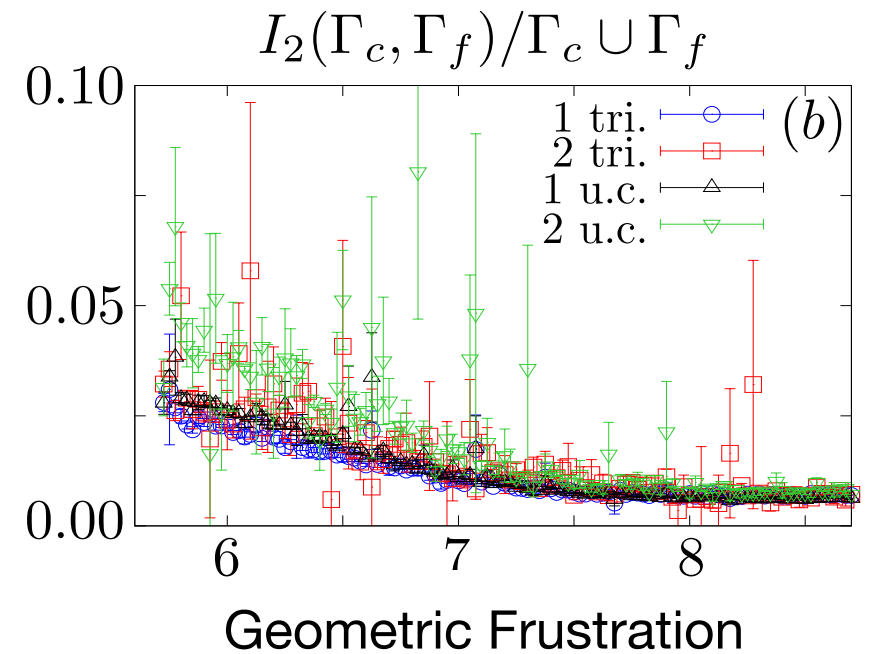
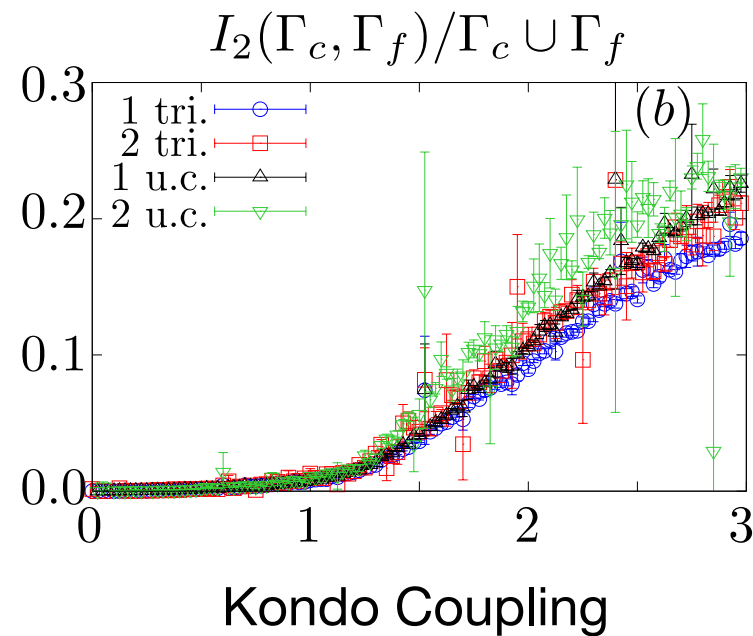
The spins enter a Z_2 spin-liquid, and decouple from the conduction electrons which form a Dirac semi-metal (no Fermi surface).



Entanglement Diagnostic of Kondo breakdown

Renyi mutual information between conduction electrons and spins:

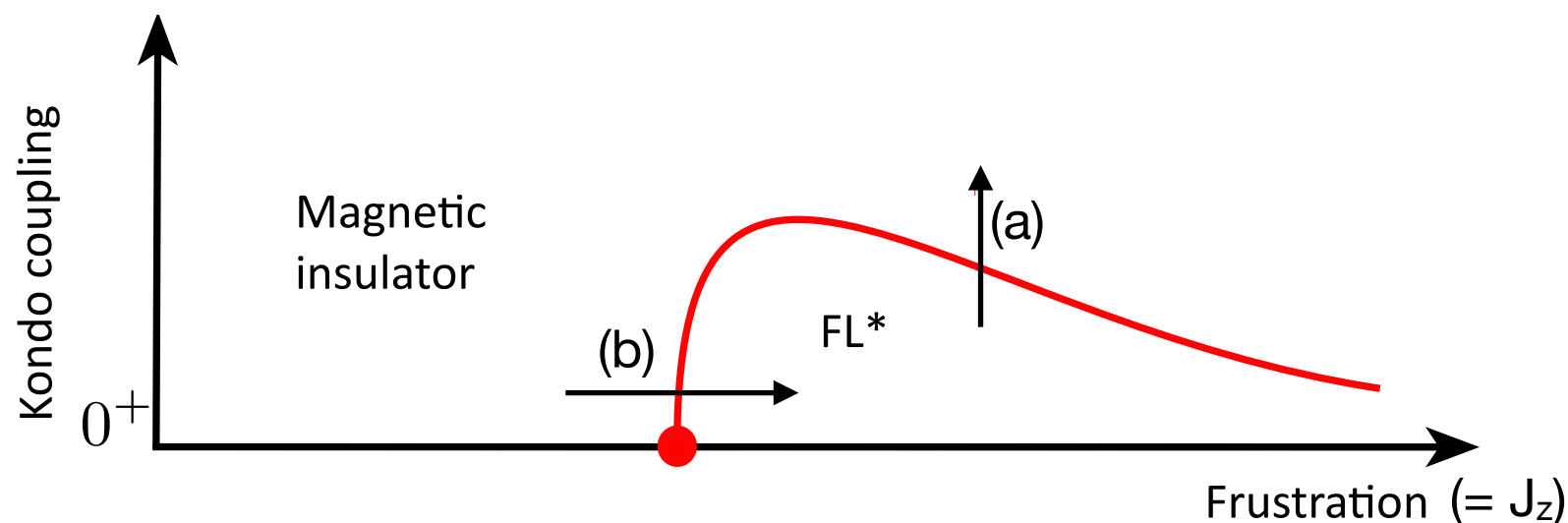
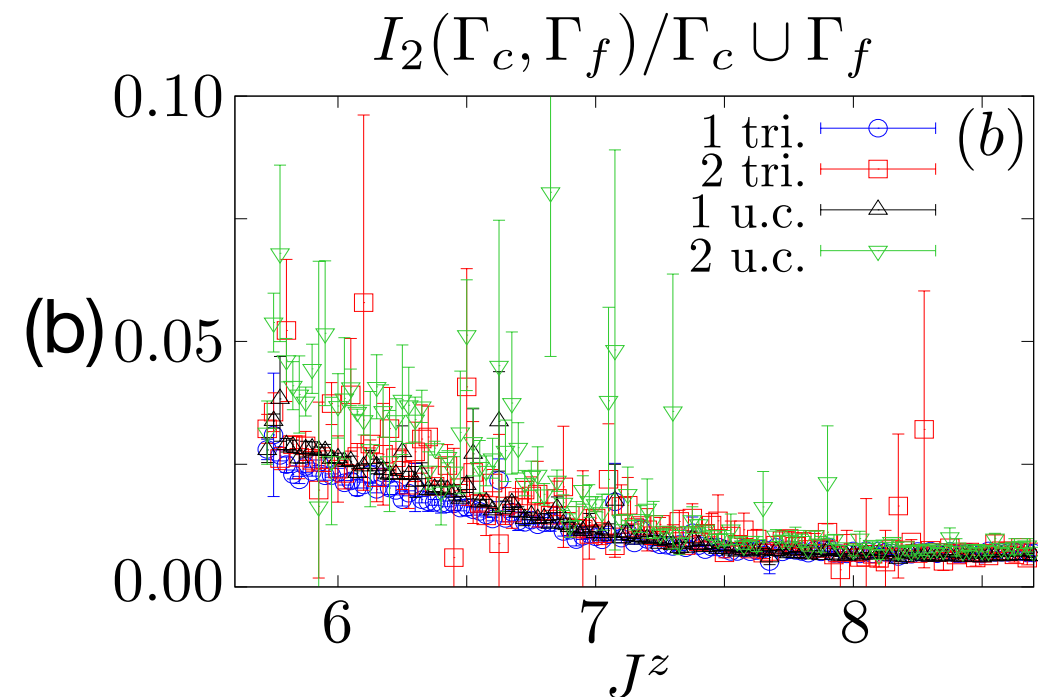
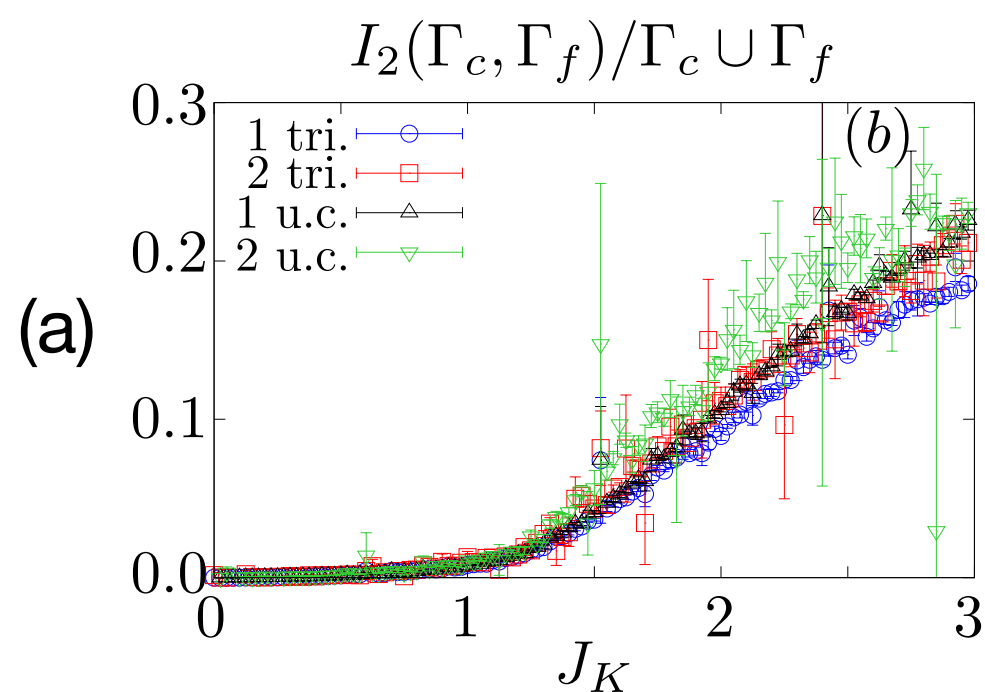
$$I_2(\Gamma_c, \Gamma_f) \equiv S_2(\Gamma_c \cup \Gamma_f) - S_2(\Gamma_c) - S_2(\Gamma_f)$$



Characterizing Kondo Screening via Entanglement

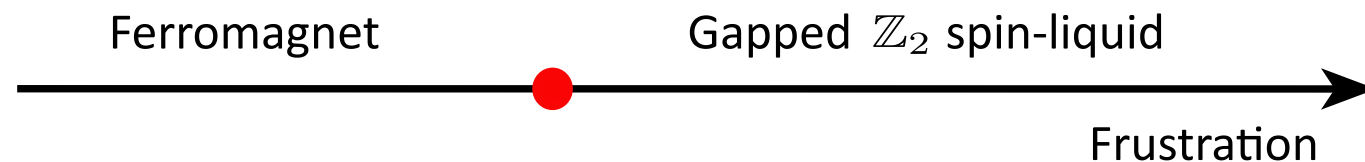
Renyi mutual information between conduction electrons and spins:

$$I_2(\Gamma_c, \Gamma_f) \equiv S_2(\Gamma_c \cup \Gamma_f) - S_2(\Gamma_c) - S_2(\Gamma_f)$$



Nature of Quantum Critical Point?

In the absence of conduction electrons, the critical point is rather unconventional, and has a rather large anomalous dimension.



$$\langle S^+(\vec{r}, \tau) S^-(0,0) \rangle \sim \frac{1}{(r^2 + \tau^2)^{1+\eta}}$$

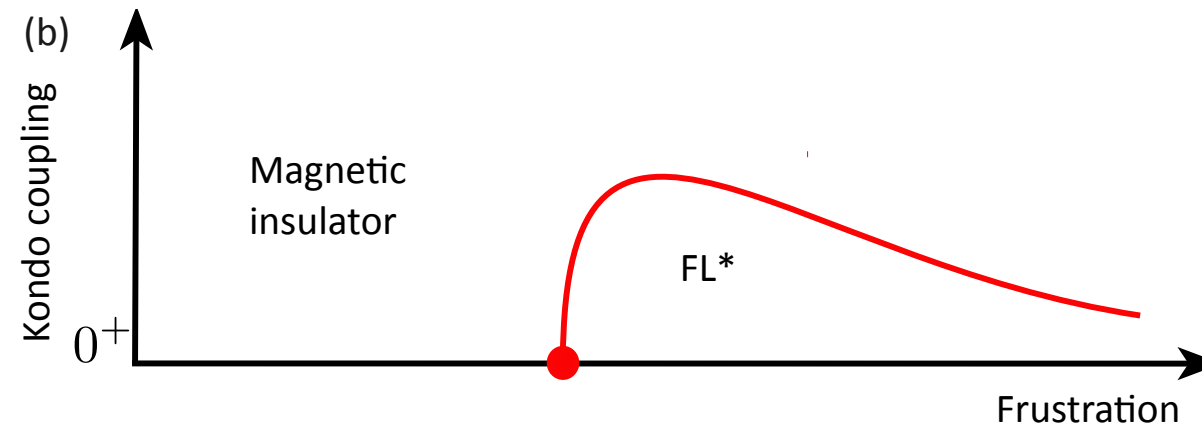
$$\eta \approx 1.37 > 1$$

(for Wilson-Fisher fixed point, $\eta \approx 0.03$)

[Chubukov, Senthil, Sachdev, 1994;
Isakov, Hastings, Melko, 2011]

Nature of Quantum Critical Point?

Kondo coupling irrelevant at the transition
⇒ Kondo breakdown even at the transition.



$$\frac{dJ_K}{dl} = (1 - \eta)J_K \quad (\langle S(r)S(0) \rangle \sim 1/r^{1+\eta})$$

Kondo coupling **irrelevant** at the critical point due to critical exponent $\eta > 1$,
a consequence of fractionalization.

Atomic spin-chain realization of a model for quantum criticality

R. Toskovic^{1†}, R. van den Berg^{2†}, A. Spinelli¹, I. S. Eliens², B. van den Toorn¹, B. Bryant¹, J.-S. Caux²
and A. F. Otte^{1*} (2016)

Magnetic cobalt adatoms on metallic copper.

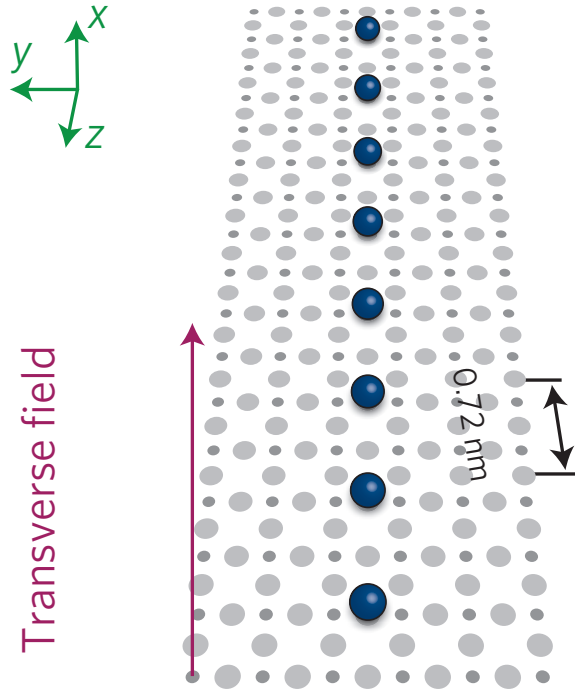
spin-3/2 Co atoms lead to effective spin-1/2 due to easy-axis anisotropy

Coupling with Cu substrate leads to an effective Kondo lattice model.

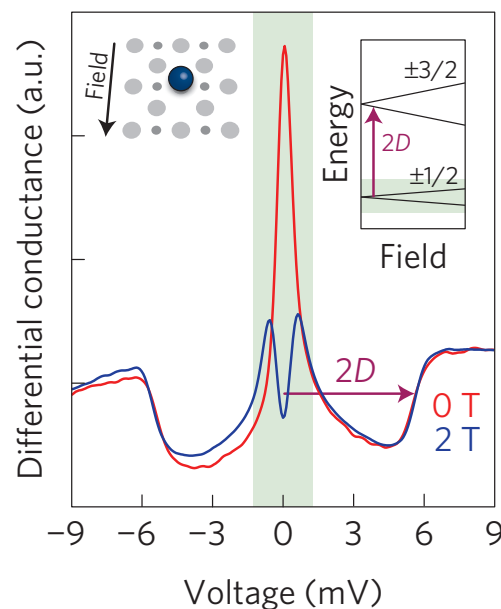
[Danu, Assaad, Mila (2019)]

$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + \text{H.c.}) + J_k \sum_{l=1}^L \hat{\mathbf{S}}_l^c \cdot \hat{\mathbf{S}}_l$$

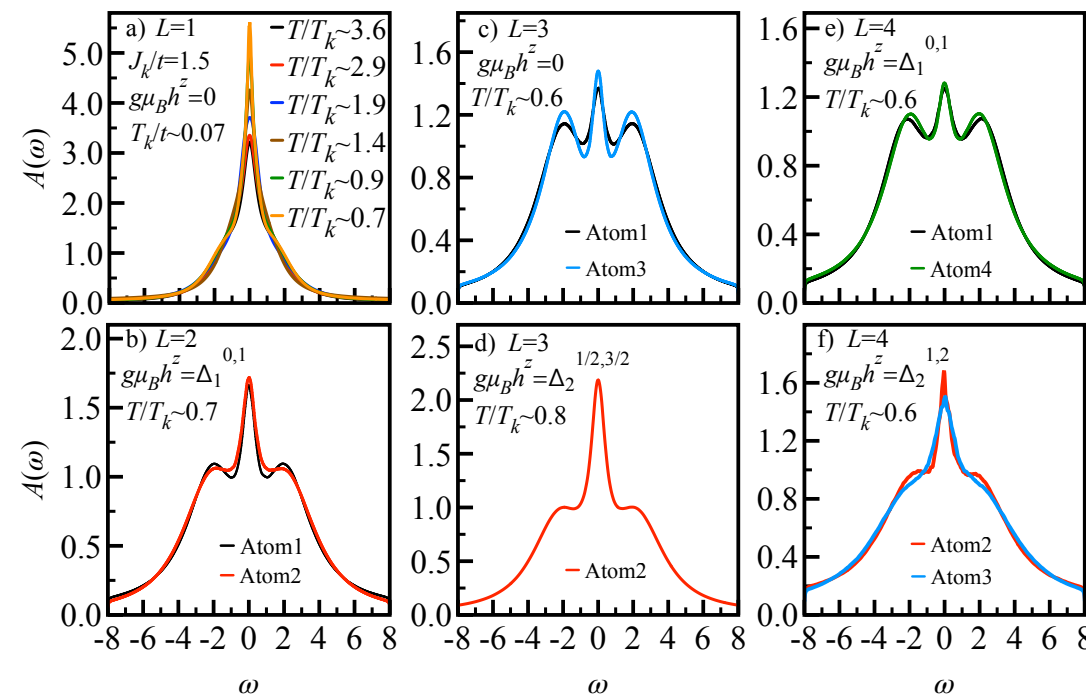
$$+ J_h \sum_{l=1}^{L-1} \hat{\mathbf{S}}_l \cdot \hat{\mathbf{S}}_{l+\Delta l} - g\mu_B h^z \sum_{l=1}^L \hat{S}_l^z.$$



(Experiment)



(QMC Simulation)



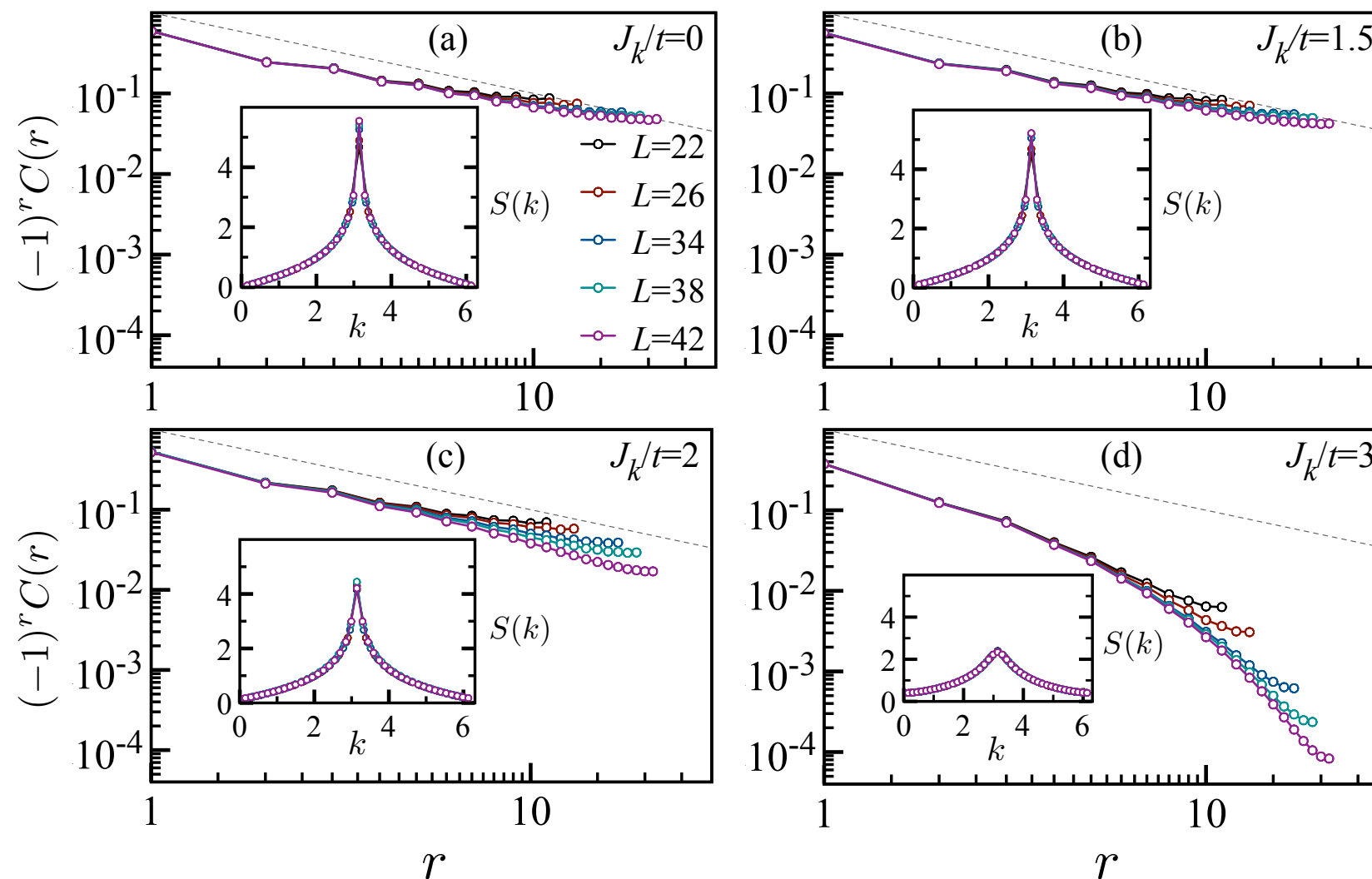
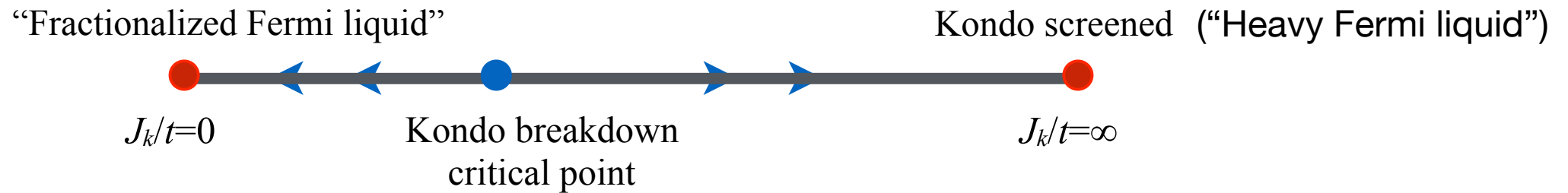
“Co-tunneling” spectral function

$$\tilde{d}_{l,\sigma}^\dagger = \hat{c}_{l,-\sigma}^\dagger \hat{S}_l^\sigma + \sigma \hat{c}_{l,\sigma}^\dagger \hat{S}_l^z$$

$$G_l^{\text{ret}}(\omega) = -i \int_0^\infty dt e^{i\omega t} \sum_\sigma \langle \{ \tilde{c}_{l,\sigma}(t), \tilde{c}_{l,\sigma}^\dagger(0) \} \rangle$$

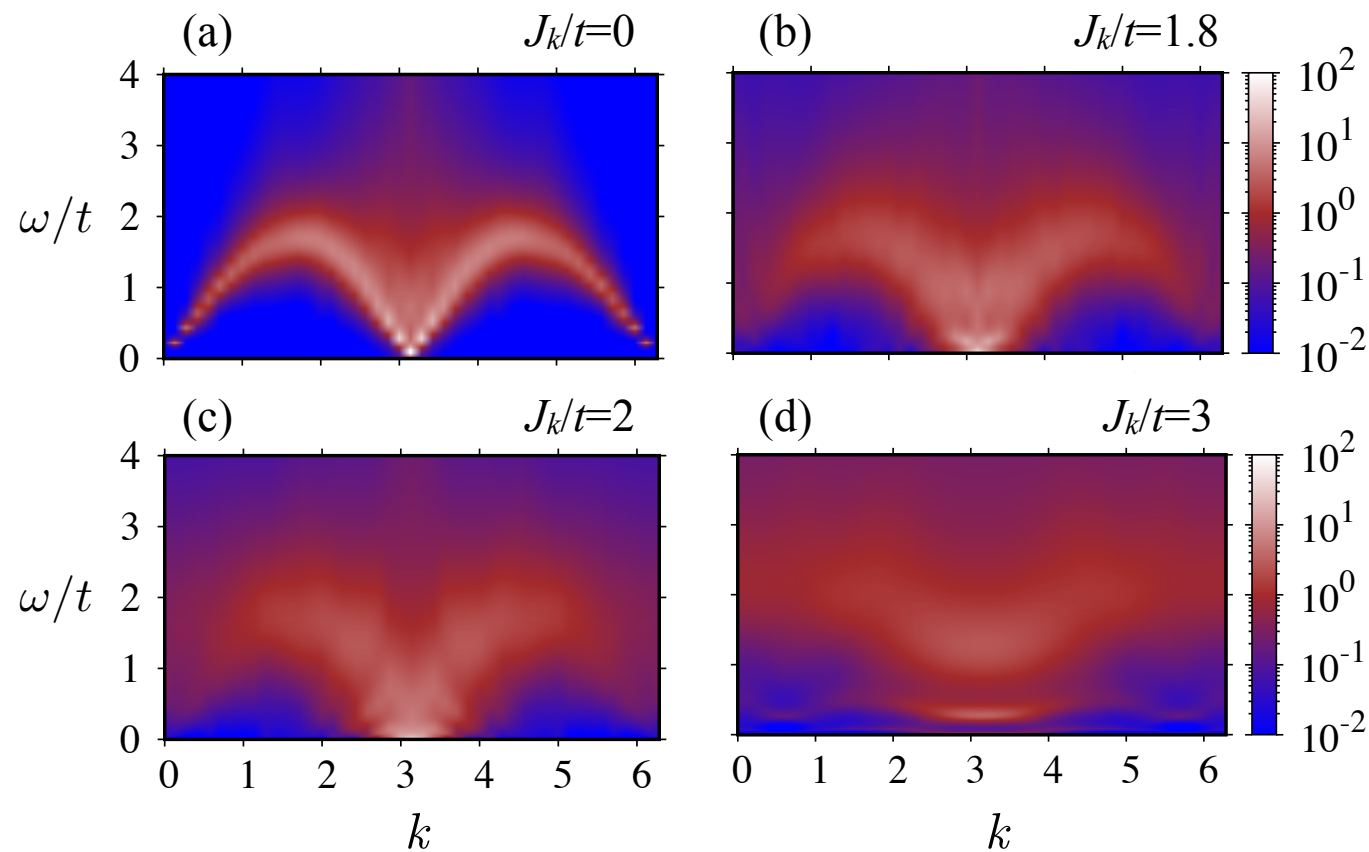
$$A_l(\omega) = -\text{Im} G_l^{\text{ret}}(\omega)$$

QMC simulations for Kondo breakdown



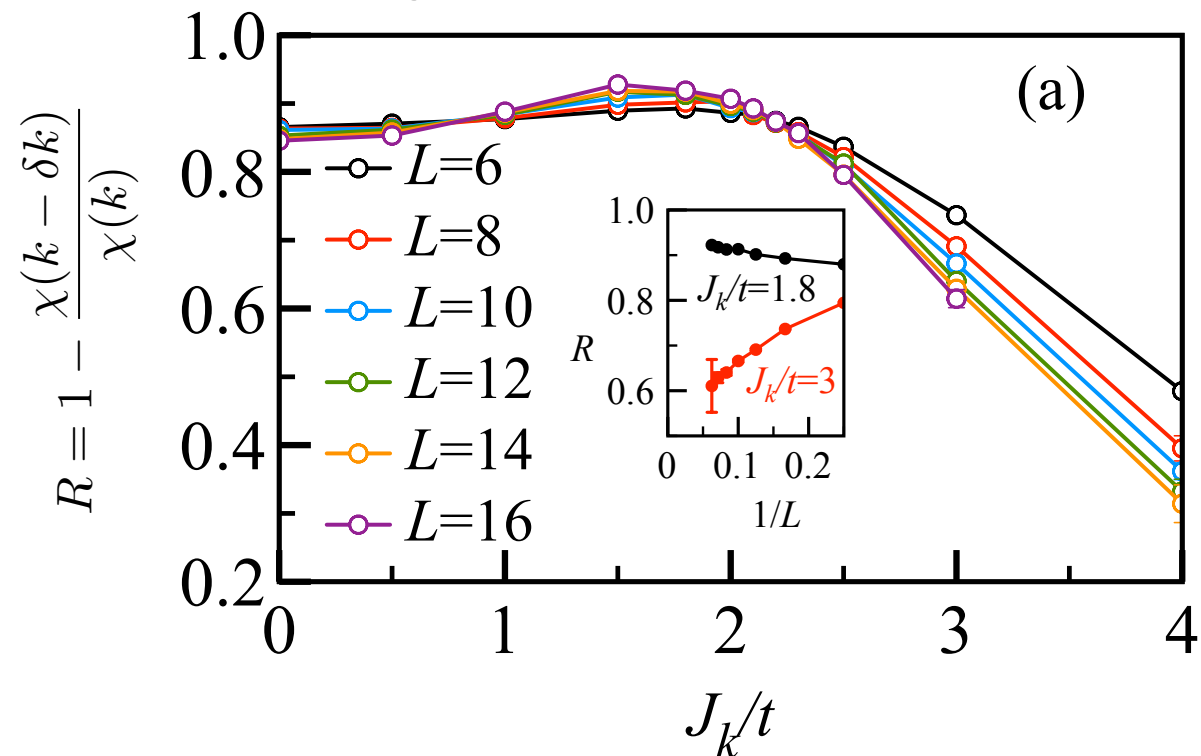
Spin-spin correlations decay as $(-1)^r \sqrt{\log(r)}/r$ for $J_K < J_{K,c}$ ($\sim$ Heisenberg chain)

and as $(-1)^r/r^4$ for $J_K > J_{K,c}$ ($\sim$ Fermi liquid)



Evidence of $z = 2$ dynamical critical exponent.

Weak signal of long-range Neel ordering in QMC ($L < 40$), possibly due to marginal relevance of dissipation



Co-tunneling (Kondo screening everywhere)

