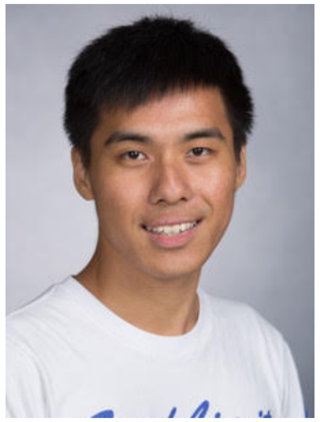
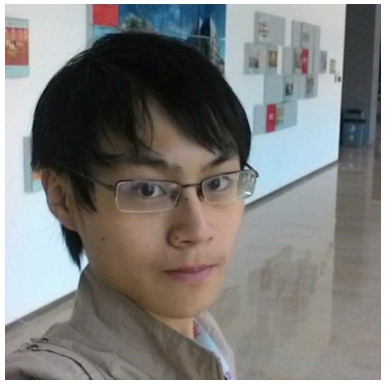
Entanglement Phase Transitions

Tarun Grover
(UCSD)



Tsung-Cheng Lu
(Perimeter)

Tim Hsieh
(Perimeter)



Kai-Hsin Wu
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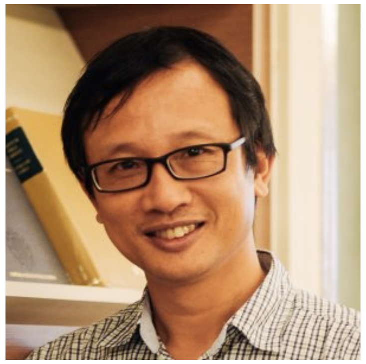


Ying-Jer Kao
(National Taiwan U.)

Information protection at thermal equilibrium

(a few examples)

Self-correcting Classical Hard-drives

2d Classical Ising model

Ferromagnet T_c Paramagnet

Self-correcting
repetition code

3d Toric code at $T > 0$

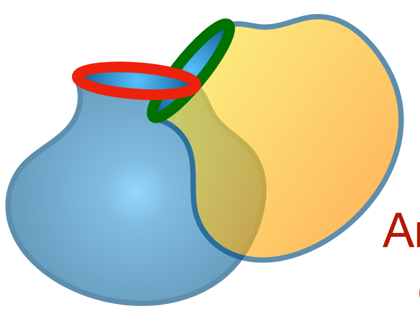
3d deconfined
Phase at $T > 0$ T_c Paramagnet

Self-correcting
topological code

Self-correcting Quantum Hard-drive

[Dennis, Kitaev, Landahl, Preskill (2001); Yoshida 2011; Hastings 2011]

4d Toric code



Anti-commuting logical
operators protected T_{c1} Only commuting logical
operators protected T_{c2} Paramagnet

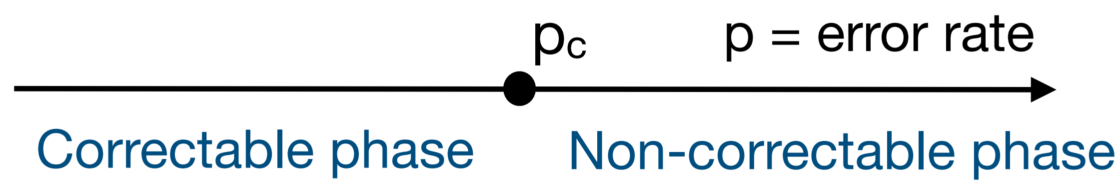
Quantum self-correcting
topological code

Classical self-correcting
topological code

Information protection out-of-equilibrium

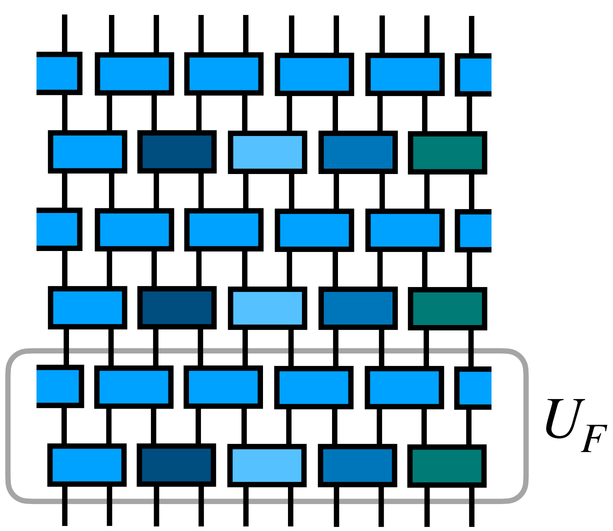
(a few examples)

Active quantum error-correction



[Shor 1996; Knill, Laflamme, Zurek 1997; Kitaev 1997;
Aharonov, Ben-Or 1999;...]

Many-body localized phase



[Basko, Aleiner, Altshuler 2005;
Oganesyan, Huse 2007, Pal, Huse 2010;...]

Many-body Quantum Zeno effect.

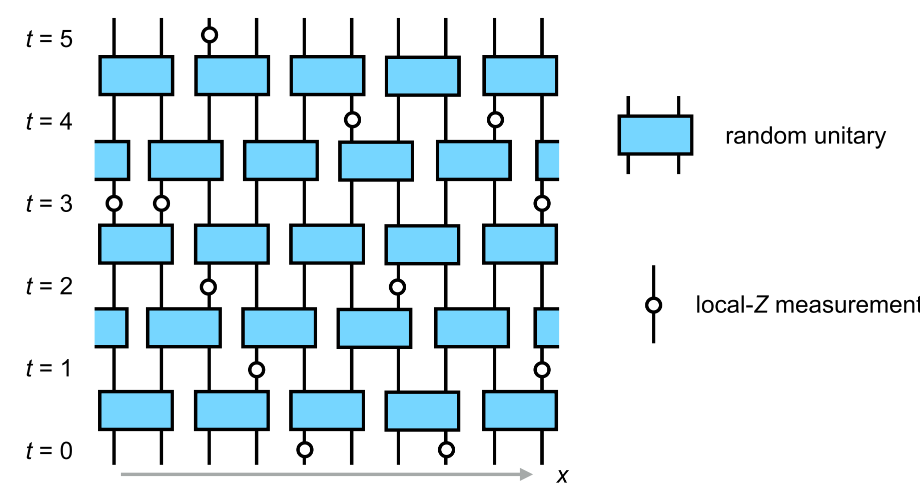


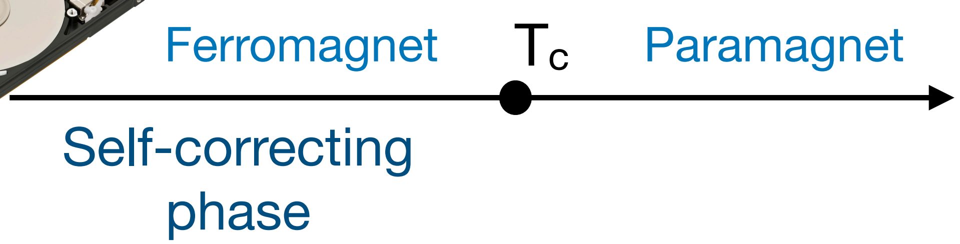
Figure from [Li, Chen, Fisher 2018]

[Li, Chen, Fisher 2018; Skinner, Ruhman, Nahum 2018;
Chan et al 2019; Gullans, Huse, 2019; Bao et al 2019; Jian et al 2019...]

Information protection and Ergodicity Breaking

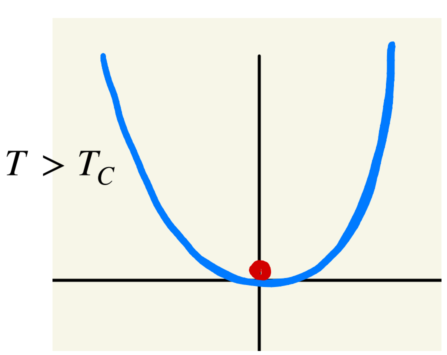
Classical hard-drive

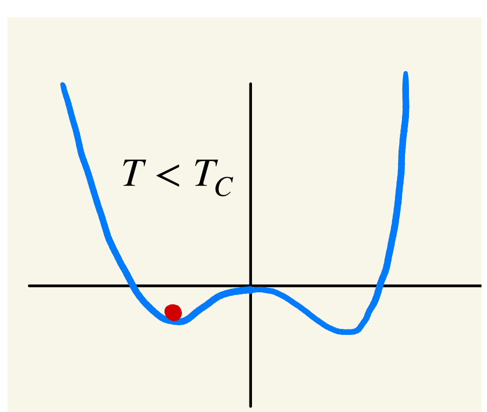
Self-correcting due to (mild) ergodicity breaking.



Can be diagnosed by a local order parameter (= magnetization)

$$\lim_{h_{SSB}\to 0^+, T=T_c^+} S - \lim_{h_{SSB}\to 0^+, T=T_c^-} S = \log(2)$$

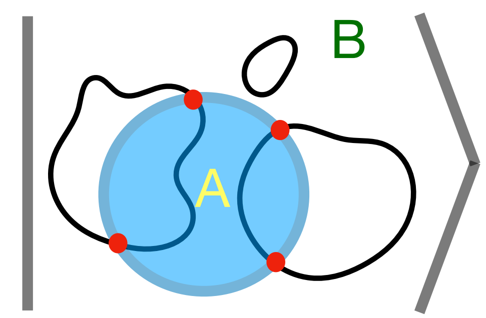




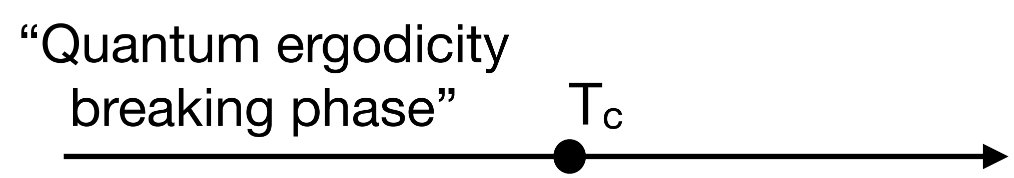
Quantum hard-drive

At T = 0, topological entanglement entropy serves as an order parameter.

What about finite T?



$$S_A = \alpha L_A - \gamma$$

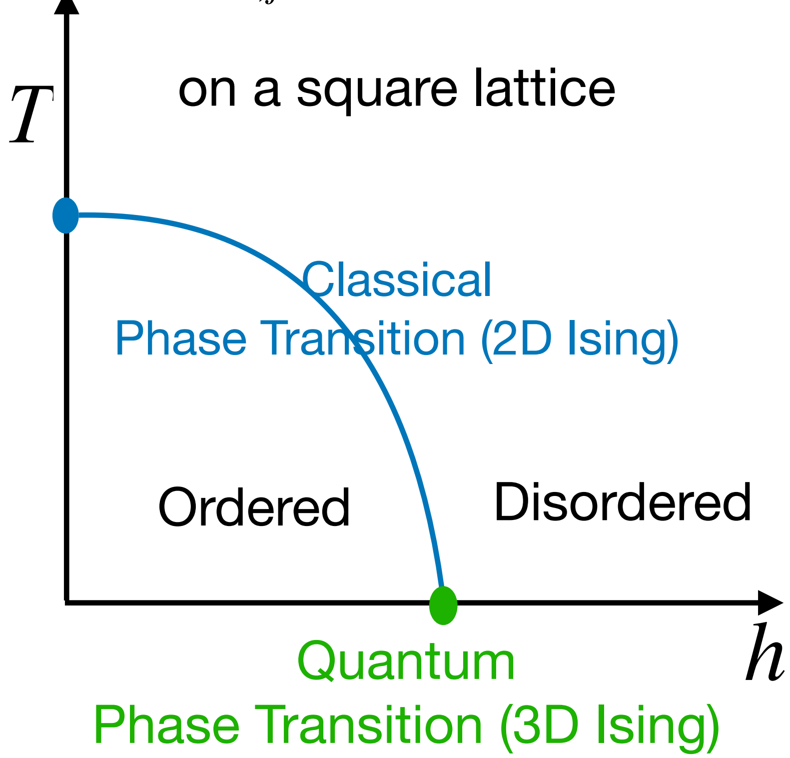


$$\lim_{T=T_c^+} ?? - \lim_{T=T_c^-} ?? \neq 0$$

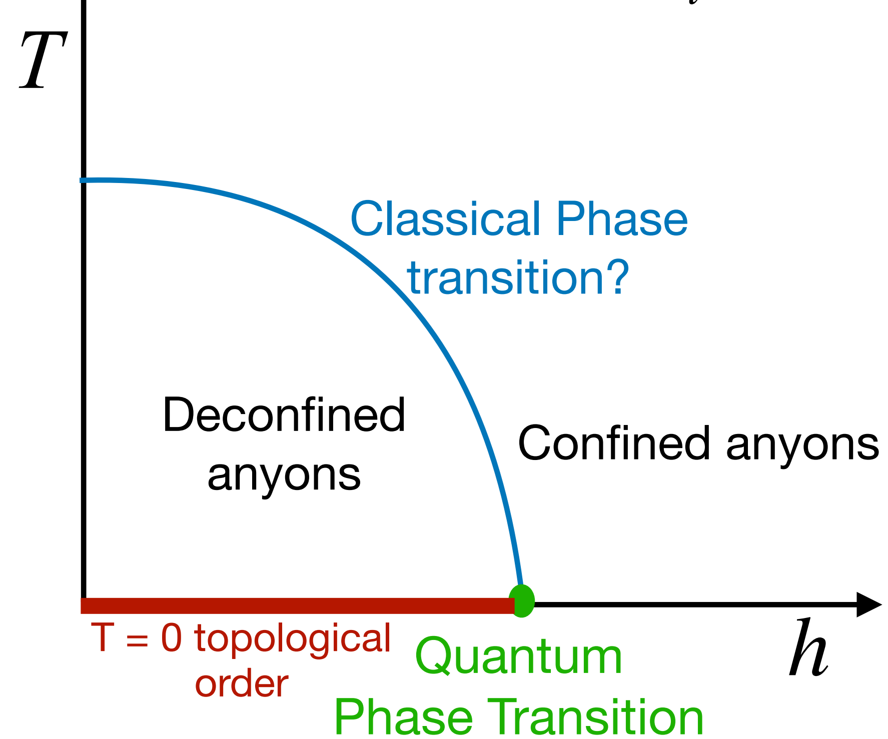
Broader question:

Distinguishing Quantum from Classical Transitions

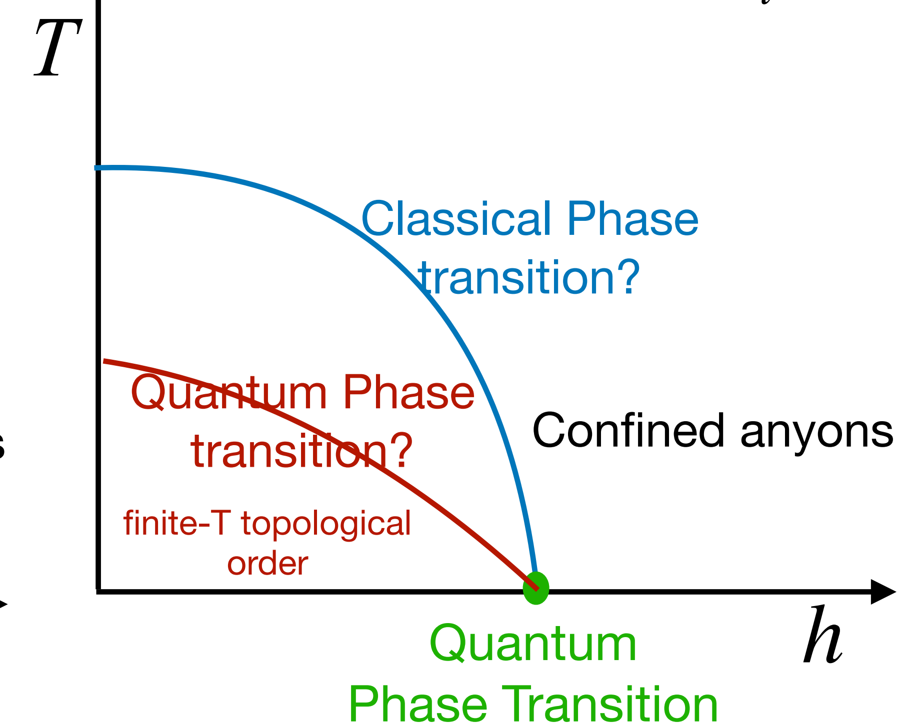
$$H = \sum_{<i,j>} -Z_i Z_j - h \sum_i X_i$$



$$H = H_{\text{3d toric code}} + h \sum_i X_i$$



$$H = H_{\text{4d toric code}} + h \sum_i X_i$$



If a finite temperature transition really classical,

quantum entanglement must be short-ranged despite a diverging correlation length.

<u>If</u> finite-T correlation length is finite,
Gibbs state can be prepared with a small depth channel.

Swingle, McGreevy 2016; Wu, Hsieh 2018; Qi, Maldacena 2018;
Brandao, Kastoryano 2019; Cottrell et al 2019, Chapman et al 2019.

If finite-T correlation length *diverges*, but the correlations fully classical, perhaps one can still prepare the corresponding Gibbs state with polynomial resources?

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Zeroth Order question:

When is a mixed state unentangled ("separable")?

Separability Criterion for Mixed States

[Werner 1989] If $\rho = \sum_i p_i \, \rho_{i,A} \otimes \rho_{i,B}$ with $p_i > 0$

in *some* basis, then the expectation value of any operator can be reproduced by an ensemble of unentangled pure states. Therefore, such states are unentangled or "separable".

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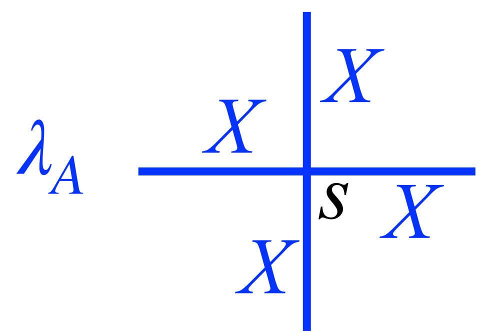
A more physical definition for local many-body systems

If $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ where each $|\psi_i\rangle$ is short-ranged entangled (⇒ no topological order or long-range

correlations), then ρ is "short-ranged entangled mixed-state".

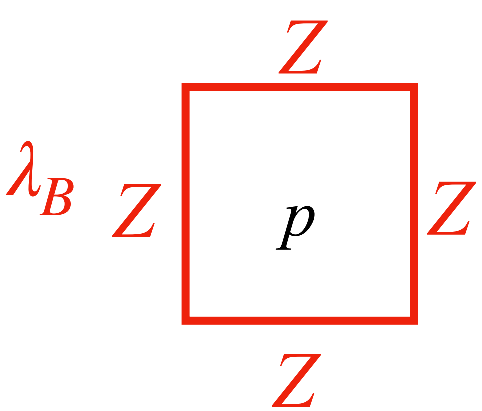
Separability in Toric codes in various dimensions

Consider 2d toric code at finite-T: $\rho = e^{-\beta H}/Z \;= e^{-\beta E_n}|E_n\rangle\langle E_n|/Z$



Each eigenstate $|E_n\rangle$ is of course topologically ordered.

Let's rewrite ρ as



$$\rho = \frac{1}{Z}e^{-\beta H} = \frac{1}{Z}e^{-\beta H/2}\sum_m |m\rangle\langle m|\, e^{-\beta H/2} = \sum_m p_m |\phi_m\rangle\langle\phi_m|$$

where $\{|m\rangle\}$ = set of all product states in the X-basis

$$|\phi_m\rangle \sim \sum_{\text{loop configs.}|C\rangle} e^{-\text{Area enclosed by loop}\times\log(\tanh(\beta\lambda_B/2))}|C\rangle$$

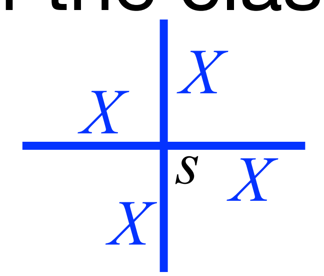
$\Rightarrow |\phi_m\rangle$ not topologically ordered at any non-zero T since large loops suppressed.

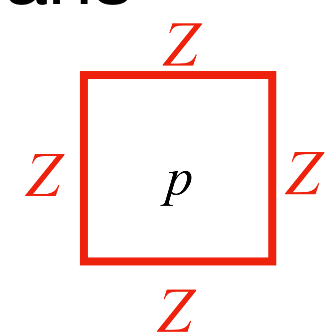
Similar arguments lead to the following conclusion

$$e^{-\beta H}/Z = \sum_m p_m |\phi_m\rangle\langle\phi_m|$$

Not topologically ordered, area-law ground-state

whenever T > min(T_A , T_B) where T_A, T_B correspond to the critical temperatures of the classical Hamiltonians

$-\lambda_A \sum_s A_s$ 

, $-\lambda_B \sum_p B_p$ 

Dimension	T_A	T_B
2D	$\frac{O(\lambda_A)}{\log L}$	$\frac{O(\lambda_B)}{\log L}$
3D	$\frac{O(\lambda_A)}{\log L}$	$O(\lambda_B)$
4D	$O(\lambda_A)$	$O(\lambda_B)$

[Tsung-Cheng Lu, Hsieh, TG 2019]

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$$e^{-\beta H}/Z = \sum_m p_m |\phi_m\rangle\langle\phi_m|$$

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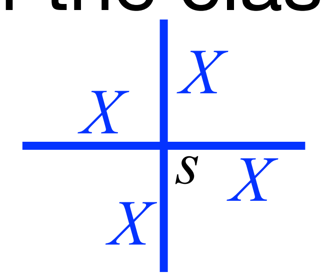
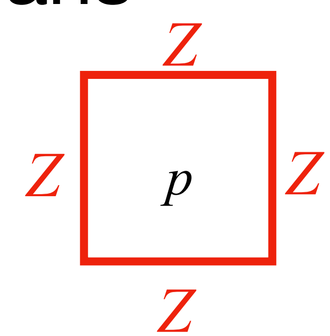
$-\lambda_A \sum_s A_s$ 

, $-\lambda_B \sum_p B_p$ 

Dimension	T_A	T_B
2D	$\dfrac{O(\lambda_A)}{\log L}$	$\dfrac{O(\lambda_B)}{\log L}$
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4D	$O(\lambda_A)$	$O(\lambda_B)$

How to quantify non-separability?

[Tsung-Cheng Lu, Hsieh, TG 2019]

Quantifying Mixed-State Entanglement

"Entanglement Negativity": $E_N = \log\left(|\rho^{T_B}|_1 \right)$ [Eisert, Plenio 1999; Vidal, Werner 2001]

- Entanglement monotone [Plenio 2005], zero for separable states (but can be zero also for non-separable states).
- Upper bounds the rate of conversion of the mixed state to Bell pairs ("distillation rate").
- Satisfies an area law for thermal states of local Hamiltonians [Sherman, Devakul, Hastings, Singh 2015].
- Calculable! (unlike most other mixed-state measures).
- Recent progress on measurement using randomized unitaries [Elben et al 2019] (tomorrow's talk).

Several condensed matter applications: Negativity of CFTs (Calabrese, Cardy, Tonni 2012), Ground state of toric code and TQFTs (Lee, Vidal 2013; Castelnovo 2013; Wen, Matsuura, Ryu 2016), Characterizing SPTs (Shapourian, Shiozaki, Ryu 2017), Probe of chaos and scrambling (Kudler-Flam et al 2019),…

Let's study non-local part of negativity as a candidate order parameter for finite-T topological order.

$$E_N = \alpha L^{d-1} - E_{N,topo}$$

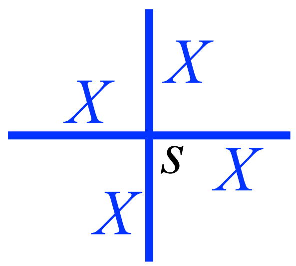
Focus on toric code in d = 2, 3, 4.

(Area-law coefficient α for 2D toric code studied in Hart, Castelnovo 2018.)

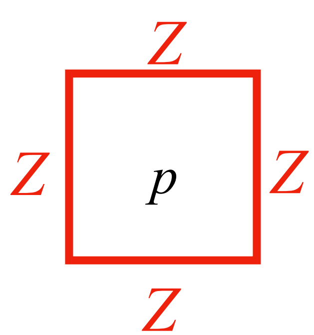
Example: 2D toric code in the limit $\lambda_B \to \infty$.

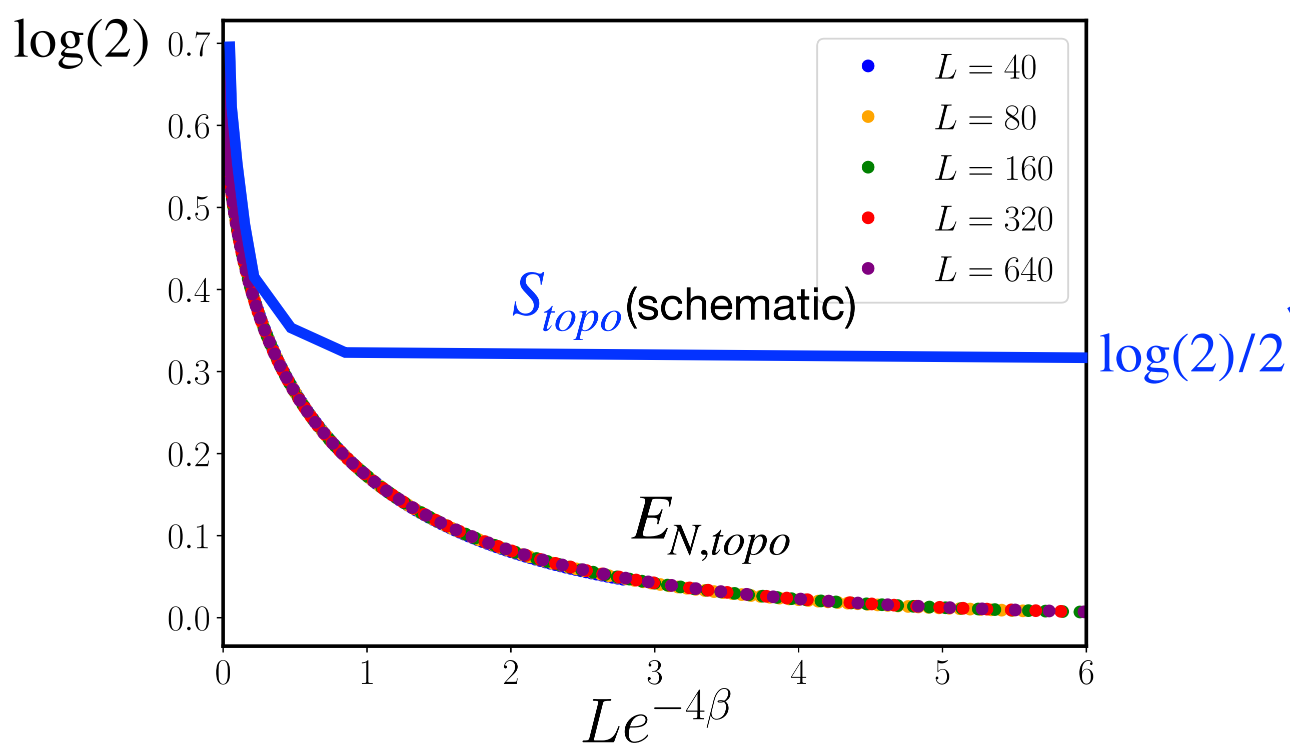
Topological negativity $E_{N,topo}$ behaves very different than the subleading contribution S_{topo} to von Neumann entropy

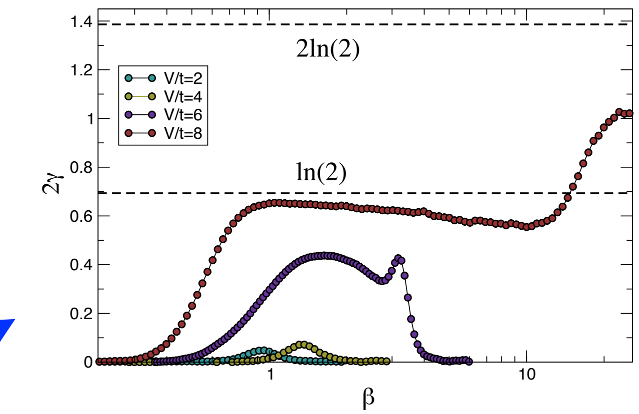
$\lambda_A = 1$



$\lambda_B \to \infty$







$$H = -t \sum_{\langle ij \rangle} [b_i^\dagger b_j + b_i b_j^\dagger] + V \sum_{\hexagon} (n_{\hexagon})^2$$

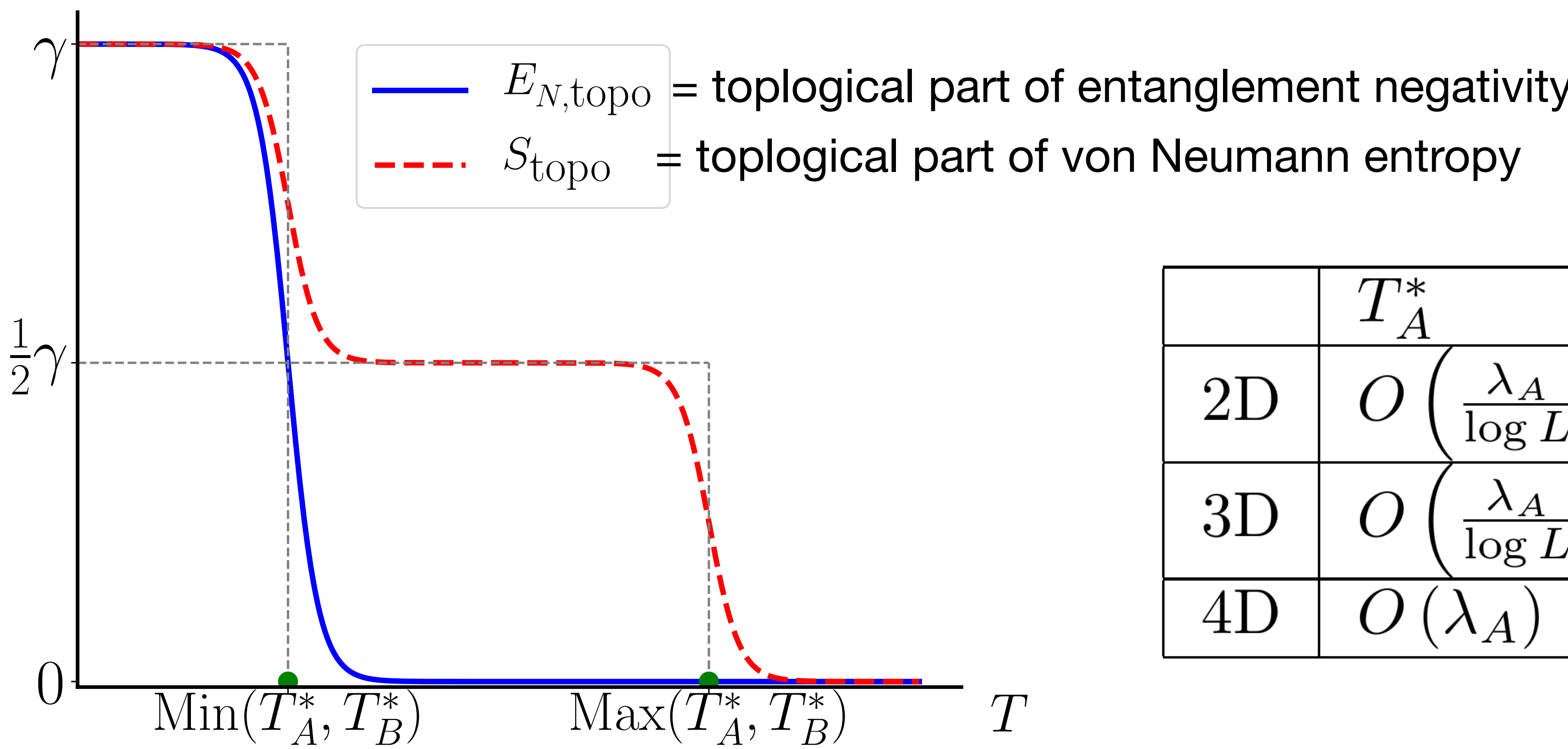
[Isakov, Hastings, Melko 2011]

$$E_{N,\mathrm{topo}} =$$

$$-\log\left\{\frac{1}{2} + \frac{\binom{L}{\frac{L}{2}+1}}{2\left(x^{1/2}+x^{-1/2}\right)^L}\left[\frac{1}{x}\ {}_2F_1\!\left(1,-\frac{L}{2}+1;\frac{L}{2}+2;-\frac{1}{x}\right) - x\ {}_2F_1\!\left(1,-\frac{L}{2}+1;\frac{L}{2}+2;-x\right)\right]\right\}$$

${}_2F_1(a,b;c;d)$: hypergeometric function $\qquad x = \tanh(\beta\lambda_A)$

Summary of results for toric code in d = 2, 3, 4



	T_A^*	T_B^*
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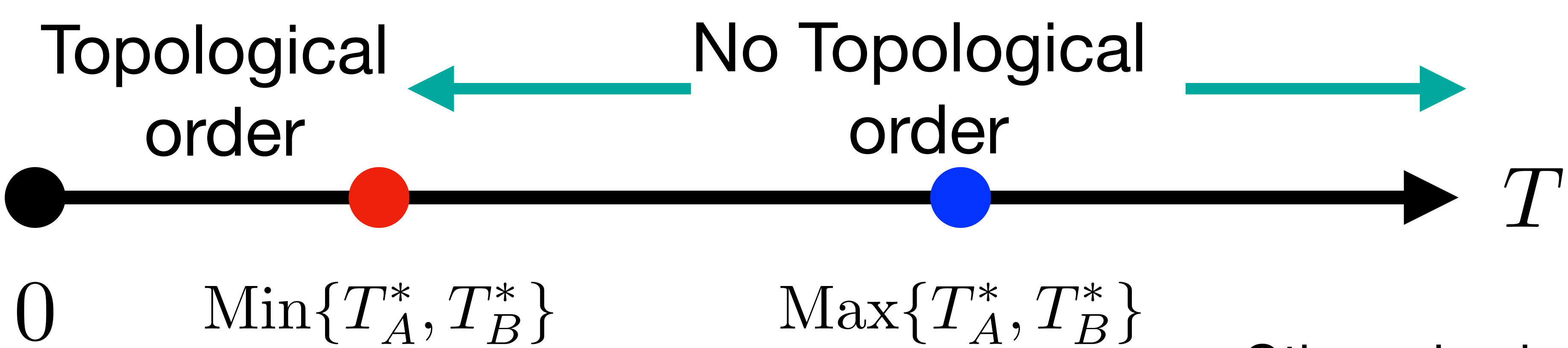


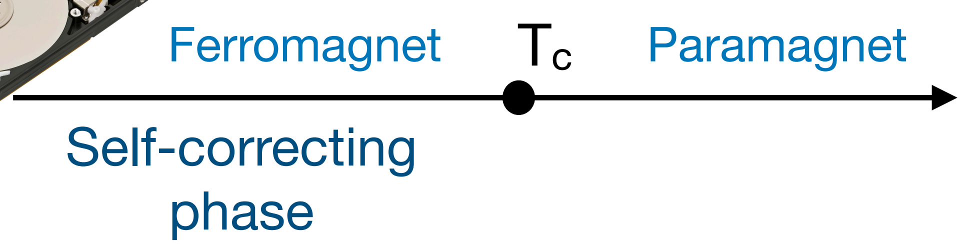
- Other mixed-state measures?
- More general topological orders?
- Field theoretic understanding?
- Quantum Monte Carlo simulations?
- Analogs for active error correction?

[Tsung-Cheng Lu, Hsieh, TG 2019]

Information protection and Ergodicity Breaking

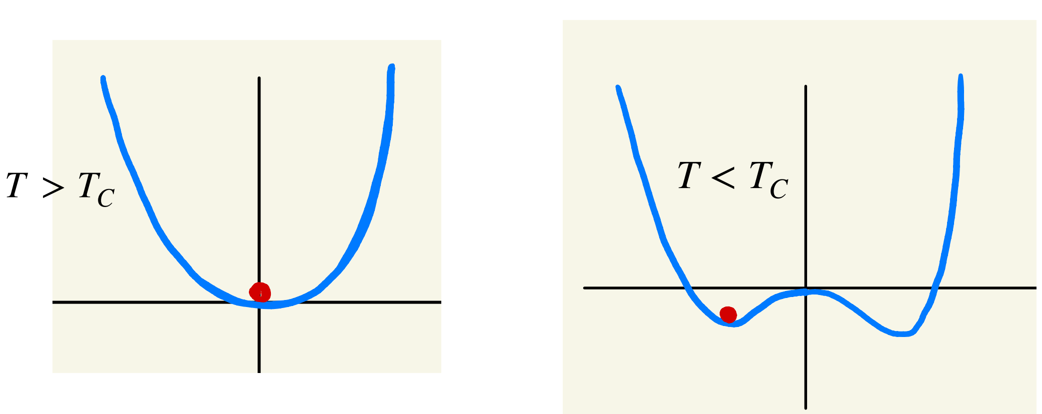
Classical hard-drive

Self-correcting due to (mild) ergodicity breaking.



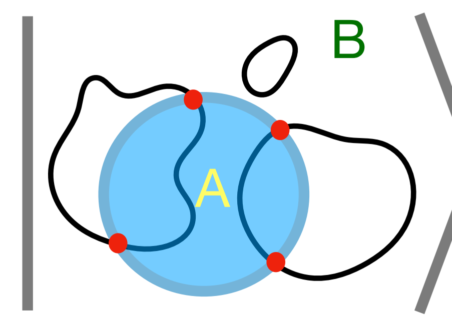
Can be diagnosed by a local order parameter (= magnetization)

$$\lim_{h_{SSB}\to 0^+, T=T_c^+} S - \lim_{h_{SSB}\to 0^+, T=T_c^-} S = \log(2)$$



Quantum hard-drive

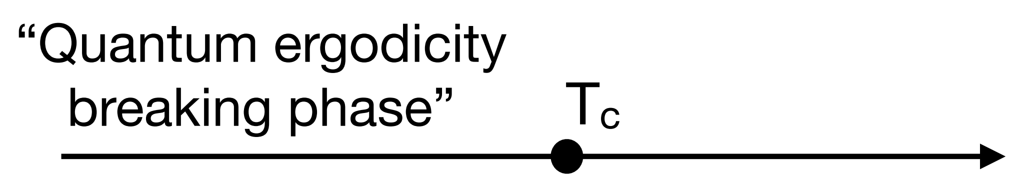
At T = 0, topological entanglement entropy serves as an order parameter.
What about finite T?



$$\boxed{\begin{aligned} S_A &= \alpha L_A - \gamma \\ \gamma &= E_{N,topo} \end{aligned}}$$

Negativity a candidate order parameter.

4d Toric code:



$$\boxed{\lim_{T=T_c^+} E_N - \lim_{T=T_c^-} E_N = 2\log(2)}$$

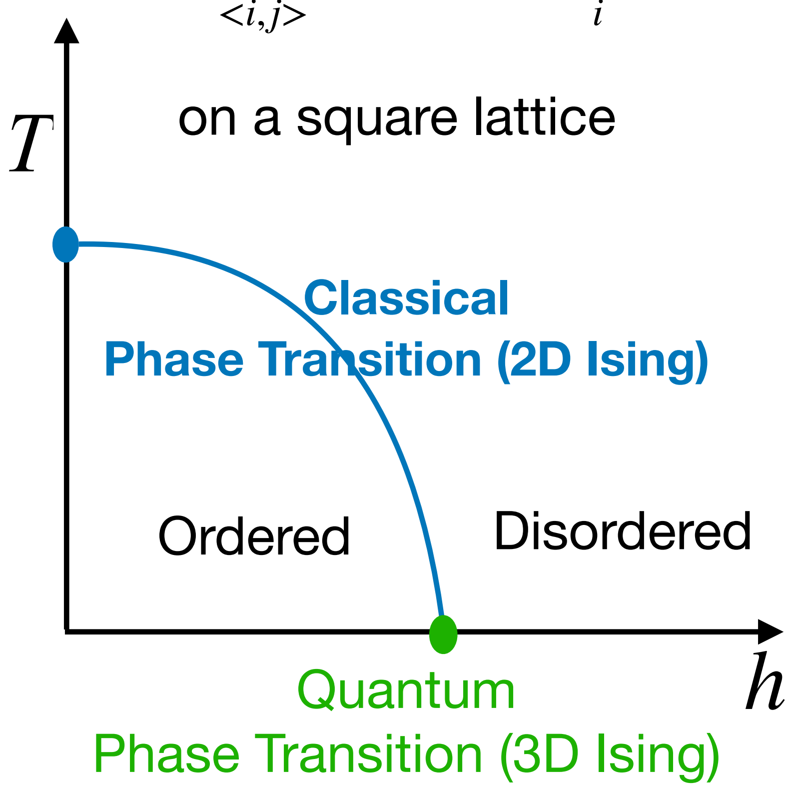
[Tsung-Cheng Lu, Hsieh, TG 2019;
More recent work: Tsung-Cheng Lu, Vijay 2022-
Relation to SPT order at the entanglement boundary]

Broader question:

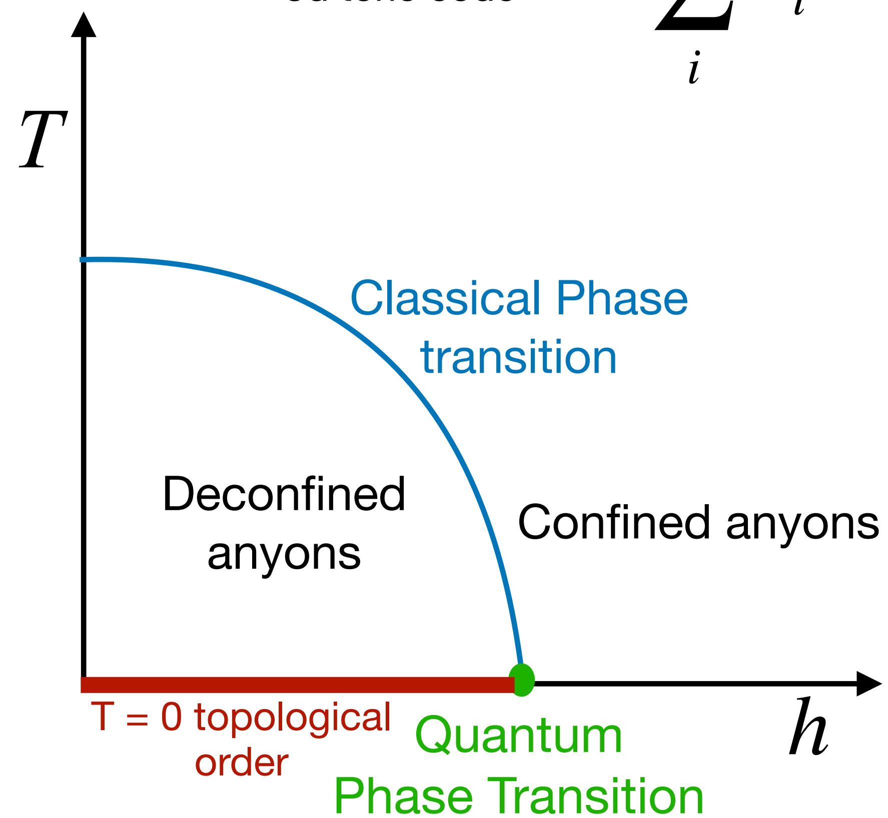
Distinguishing Quantum from Classical Transitions

$$H = \sum_{<i,j>} -Z_i Z_j - h \sum_i X_i$$

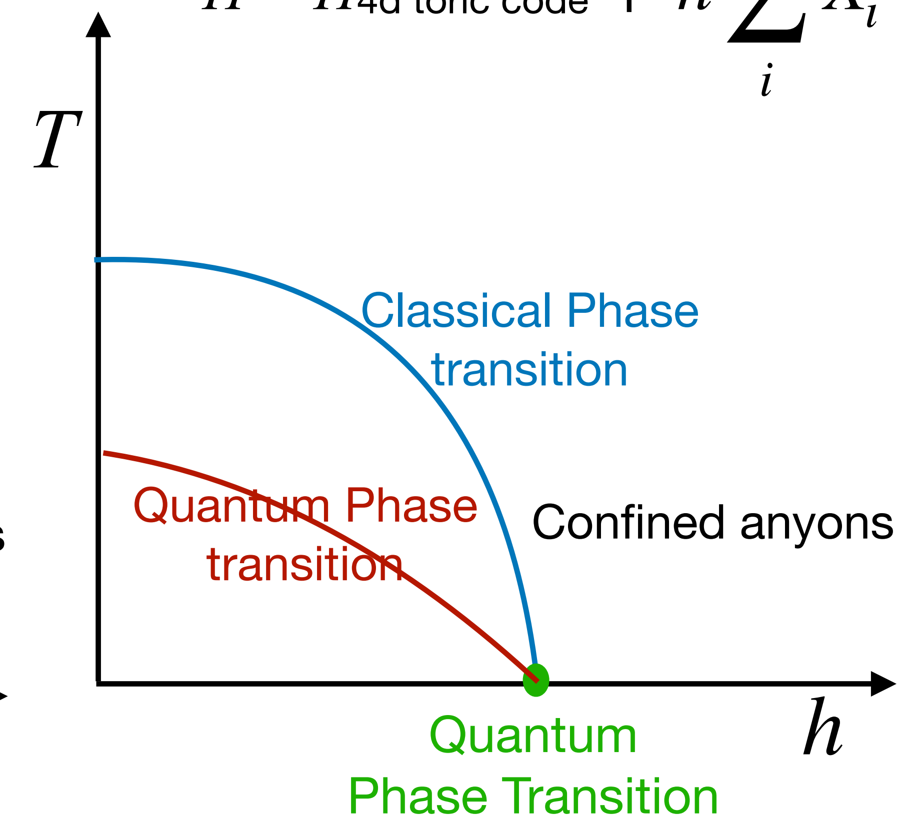
on a square lattice



$$H = H_{\text{3d toric code}} + h \sum_i X_i$$



$$H = H_{\text{4d toric code}} + h \sum_i X_i$$



If finite temperature transition really classical,

quantum entanglement must be short-ranged despite a diverging correlation length.

Consider the transverse field Ising model on square lattice…

$$H = \sum_{<i,j>} -Z_i Z_j - h \sum_i X_i$$

Although we can't calculate negativity $E_N = \log \left(\left| \rho^{T_B} \right|_1 \right)$

one can calculate a closely related quantity in Quantum Monte Carlo:

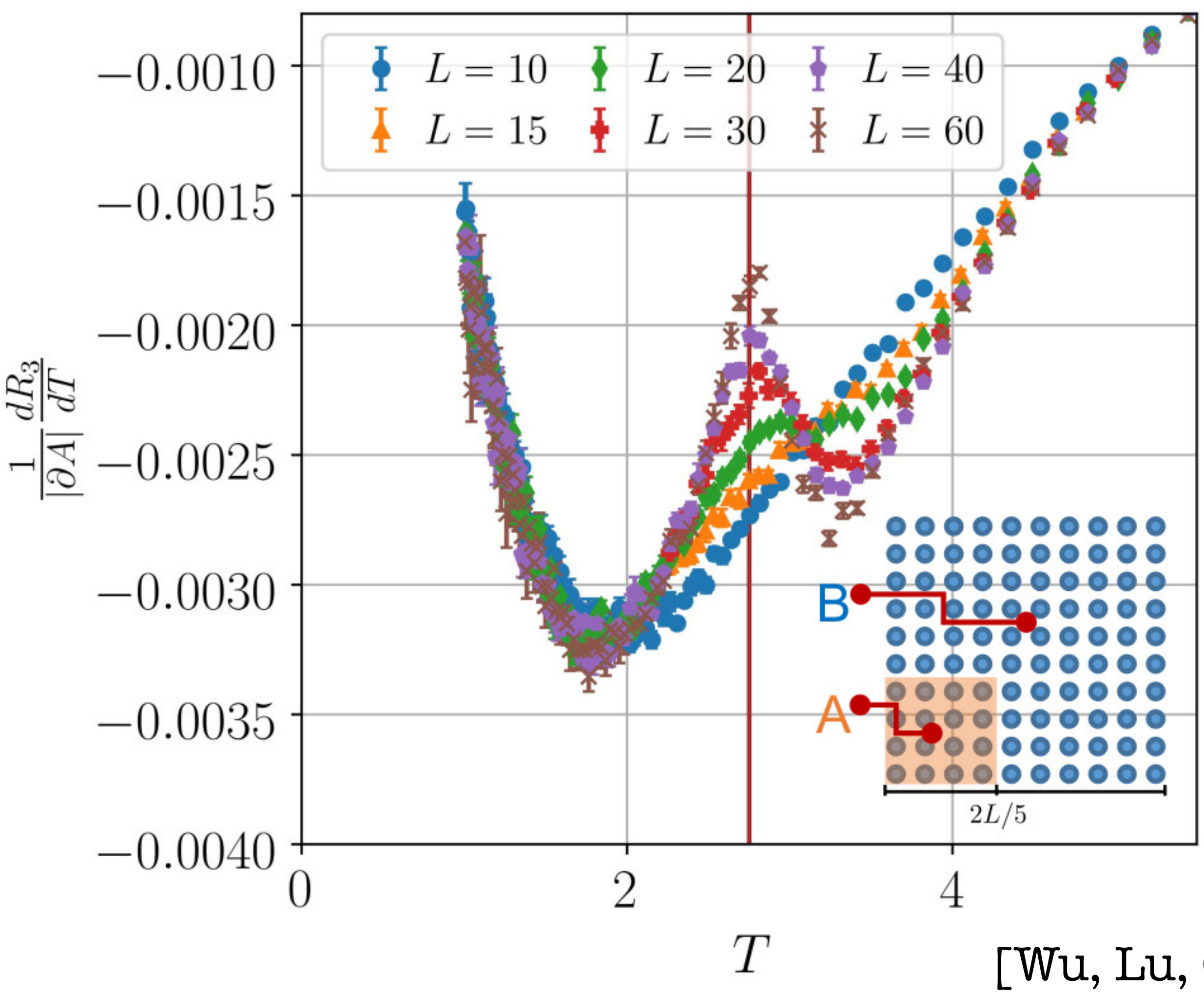
$$\tilde{R}_3 = \log \left(\frac{\text{trace} \left(\rho^{T_B} \right)^3}{\text{trace} \left(\rho^3 \right)} \right)$$

Not an entanglement measure, but in 1+1-D CFTs, has same scaling as negativity.

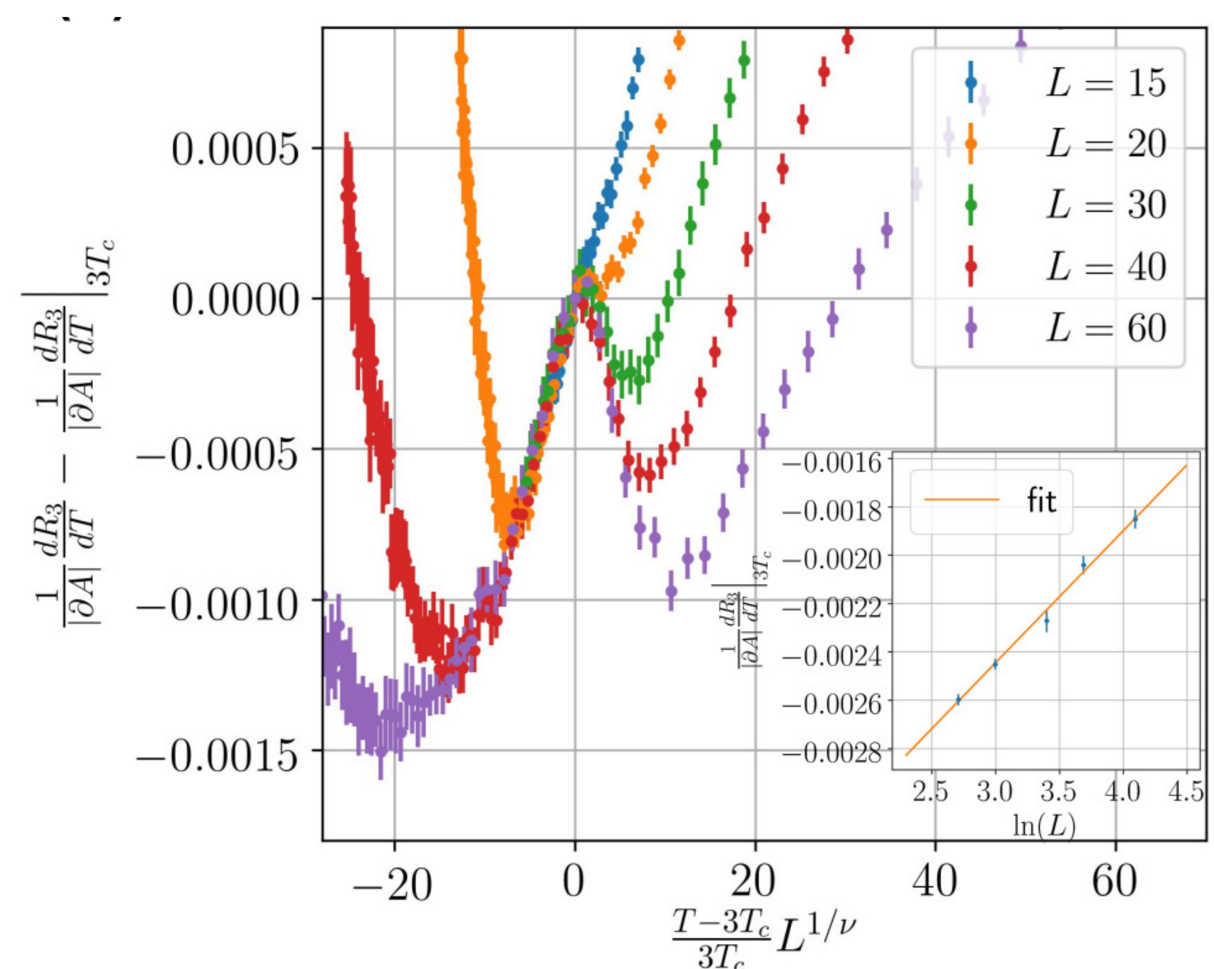
[Calabrese, Cardy, Tonni 2012]

Singularity in the area-law coefficient

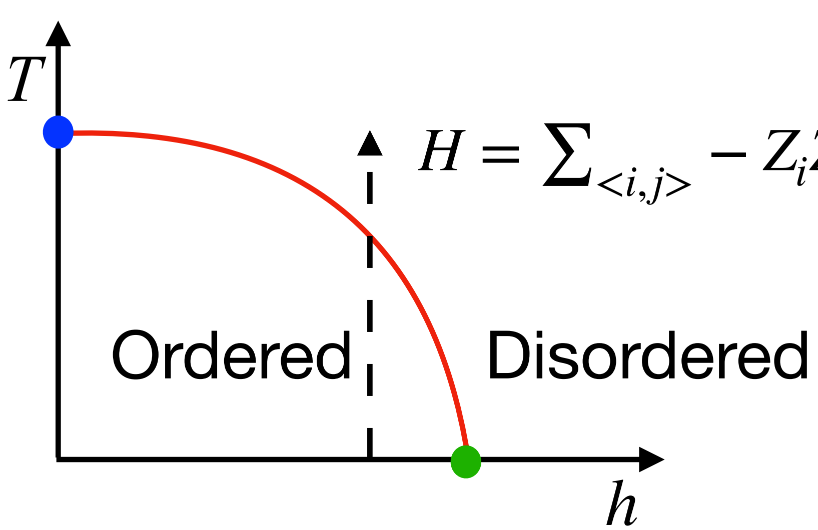
Temperature derivative of Renyi negativity



Scaling collapse



[Wu, Lu, Chung, TG, Kao 2019]

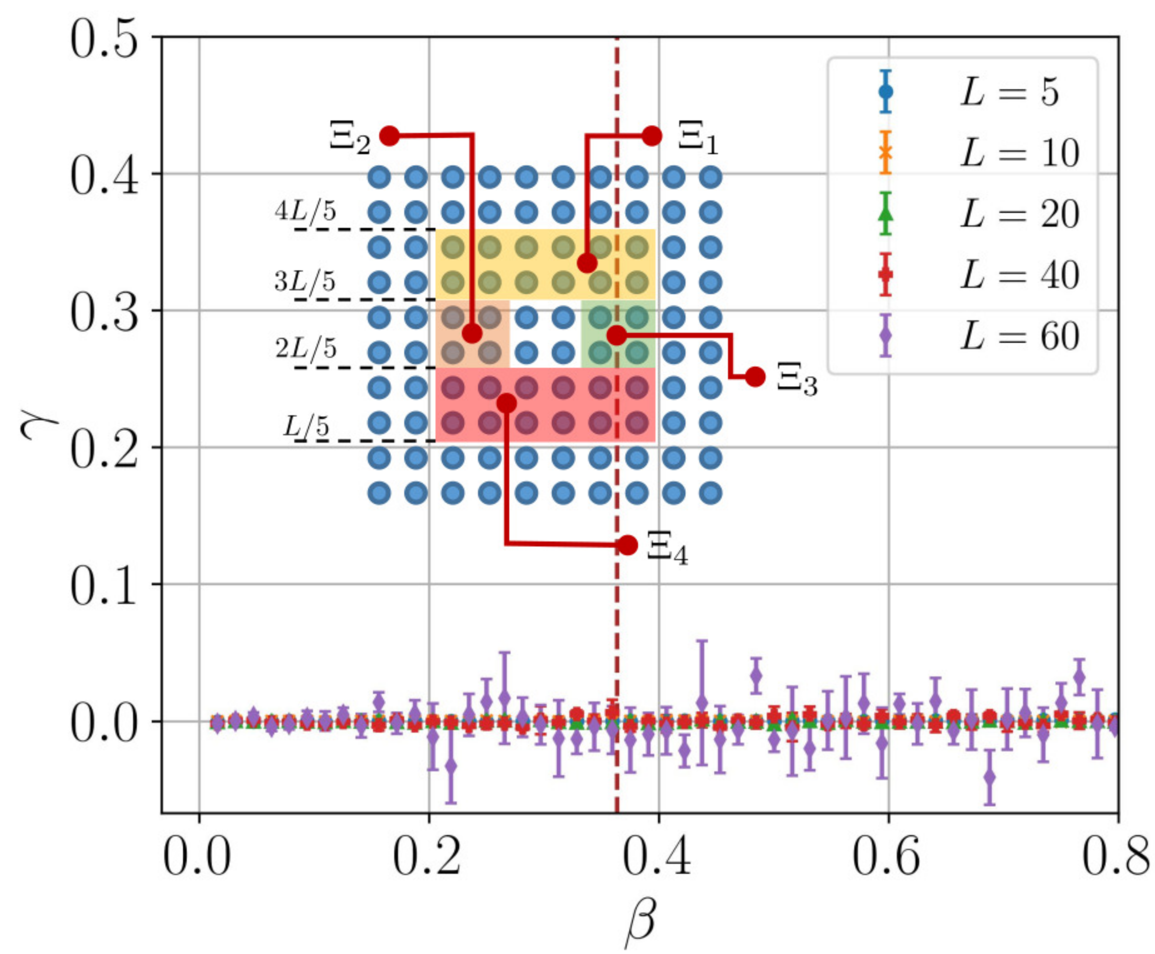


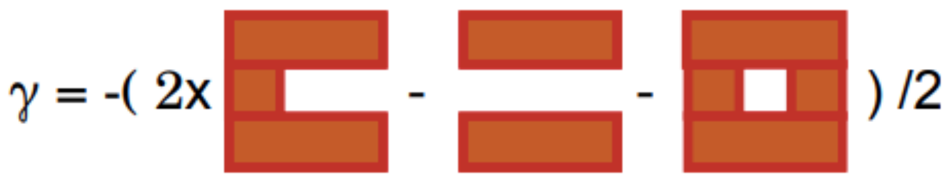
$$H = \sum_{<i,j>} -Z_i Z_j - h\sum_i X_i$$

Entropy: For $T < T_c$, $S(h_{SSB} = 0) - \lim_{h_{SSB}\to 0^+} S = \log(2)$

Negativity: For $0 < T < T_c$, $E_N(h_{SSB} = 0) - \lim_{h_{SSB}\to 0^+} E_N = 0$

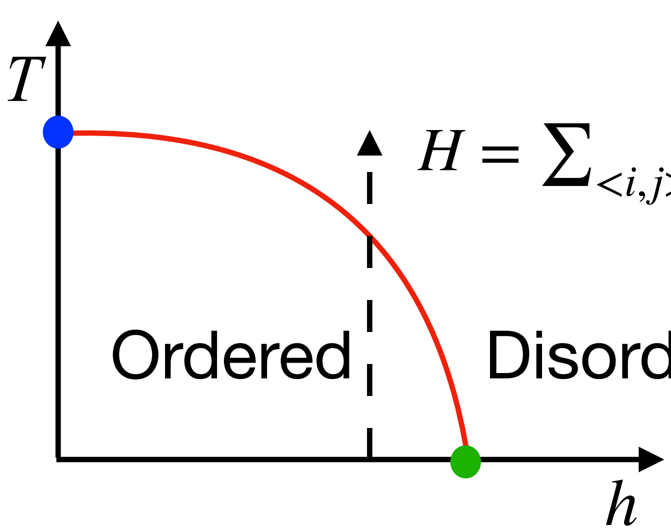
Long-range part of Renyi negativity





(similar to Levin-Wen scheme to extract topological EE).

[Wu, Lu, Chung, TG, Kao 2019]



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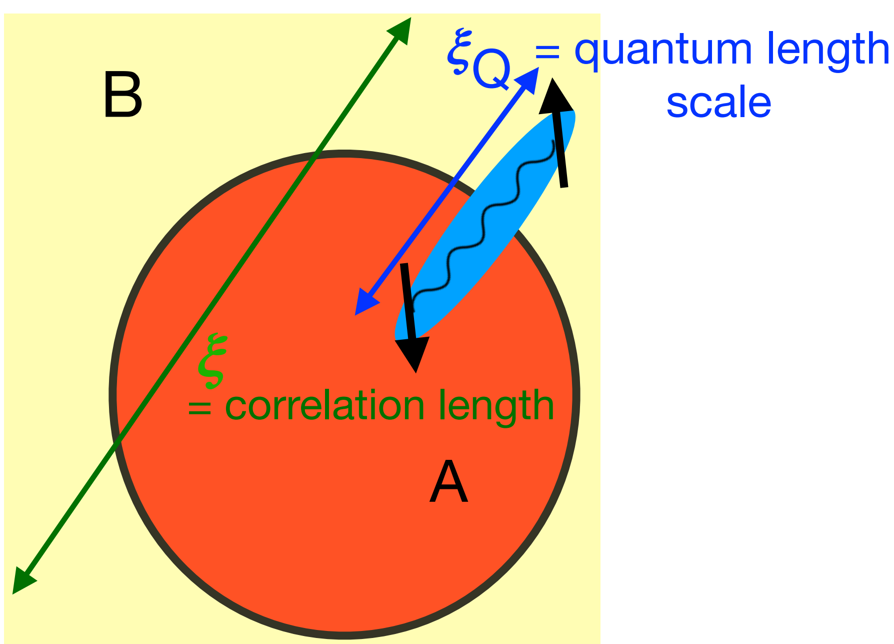
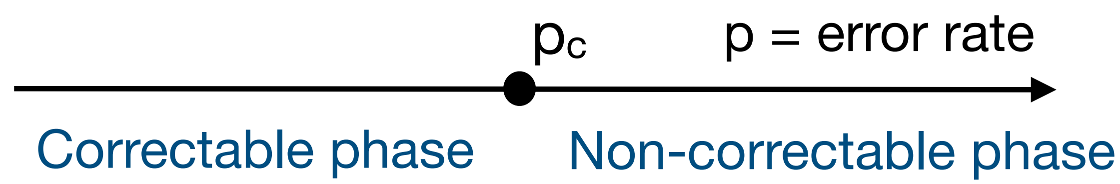


Information protection out-of-equilibrium

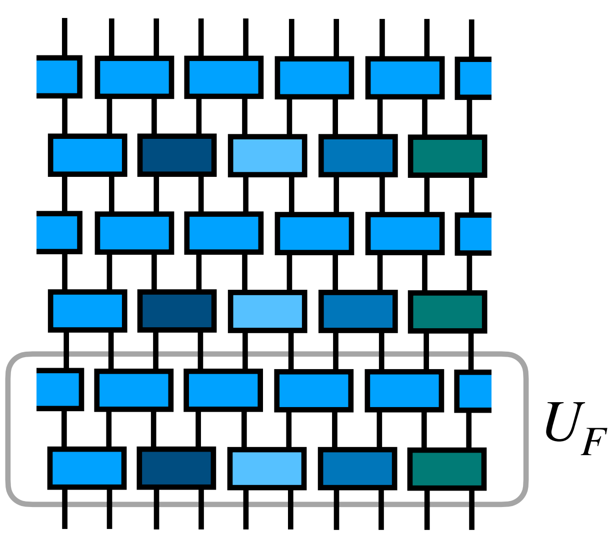
(a few examples)

Active quantum error-correction



[Shor 1996; Knill, Laflamme, Zurek 1997; Kitaev 1997; Aharonov, Ben-Or 1999;...]

Many-body localized phase



[Basko, Aleiner, Altshuler 2005; Oganesyan, Huse 2007, Pal, Huse 2010;...]

Many-body Quantum Zeno effect.

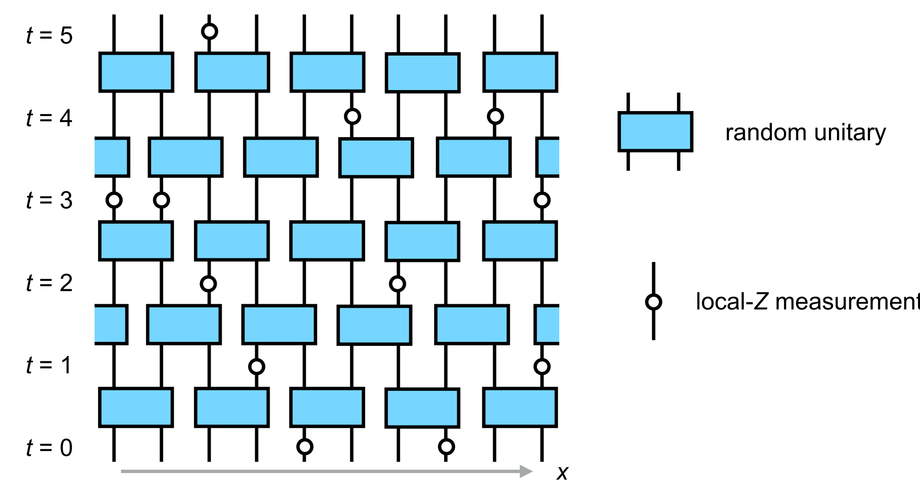


Figure from [Li, Chen, Fisher 2018]

[Li, Chen, Fisher 2018; Skinner, Ruhman, Nahum 2018; Chan et al 2019; Gullans, Huse, 2019; Bao et al 2019; Jian et al 2019...]

Broader question: Phases of Quantum Dynamics?

Infinitely rich variety of quantum phases of matter at T = 0.

Fermi liquids, superconductors, quantum Hall phases, Mott insulators, …

How rich is the landscape of time-evolved many-body states?

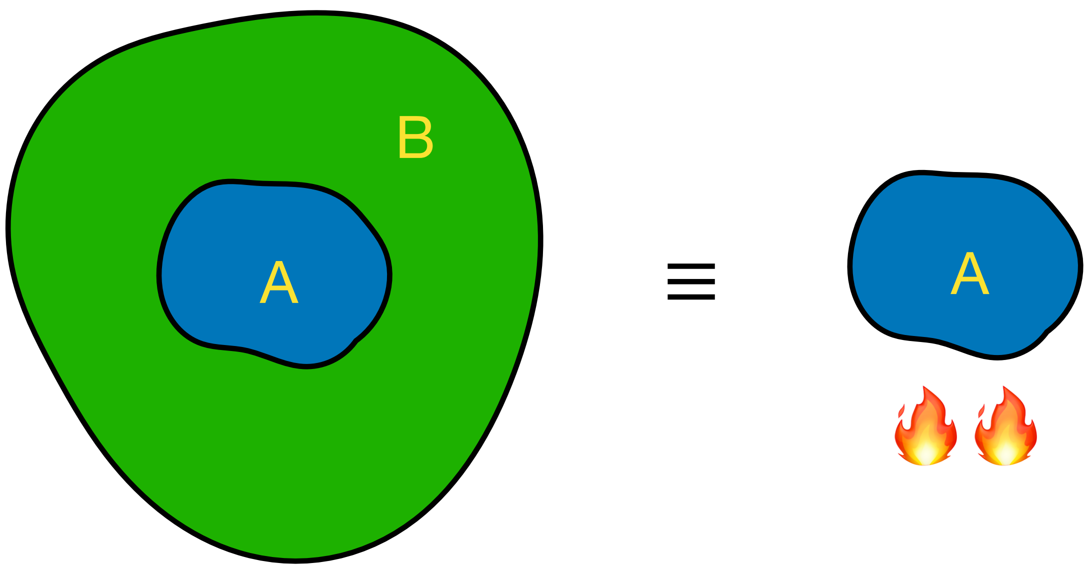
$$|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$$

Unfortunately, quantum chaos is an obstacle to find novel phases of quantum dynamics.

Broader question: Phases of Quantum Dynamics?

$$|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$$

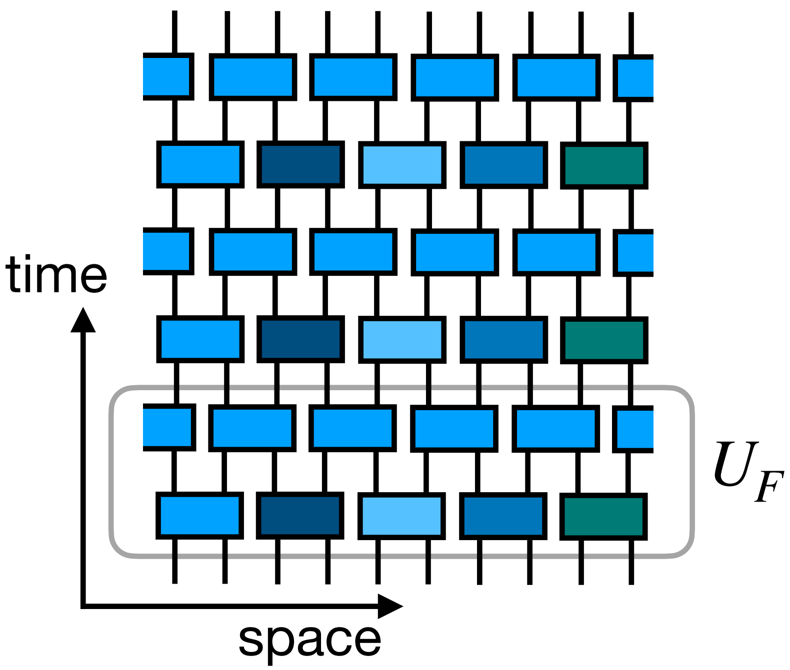
If a many-body system is chaotic, a small subsystem behaves as if it was in thermal equilibrium. [Deutsch 1991; Srednicki 1994]



To obtain new physics non-thermal physics, one needs to impede entanglement growth.

How to obstruct entanglement growth?

One route: spatial disorder.

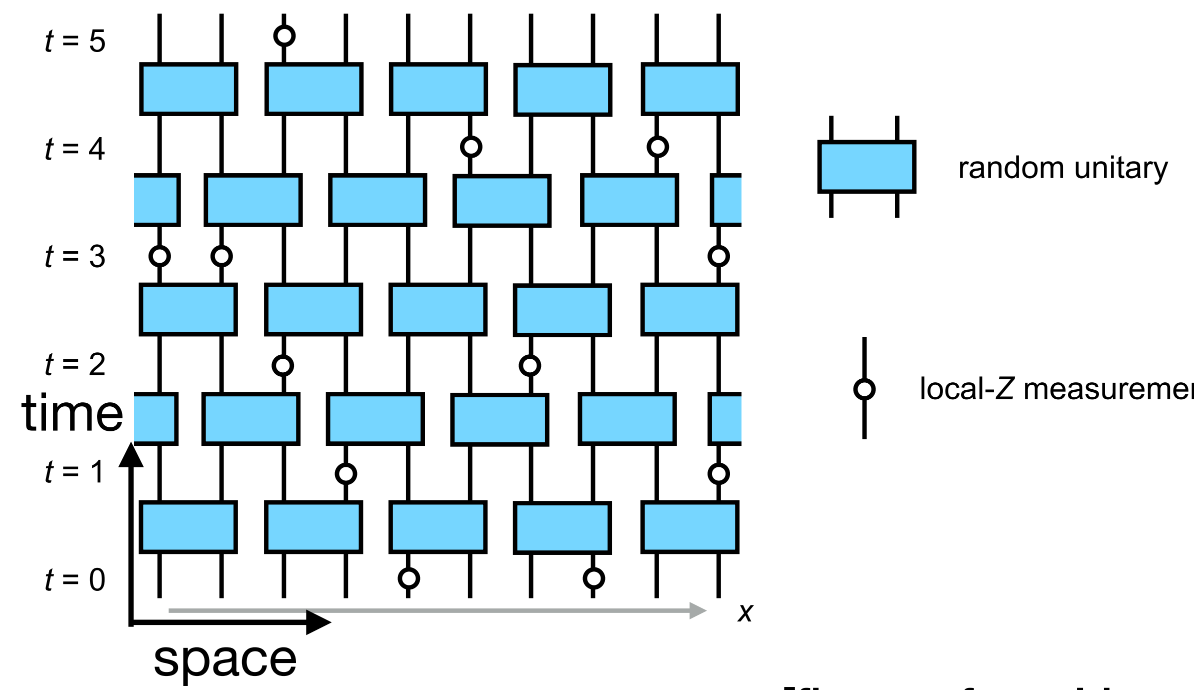


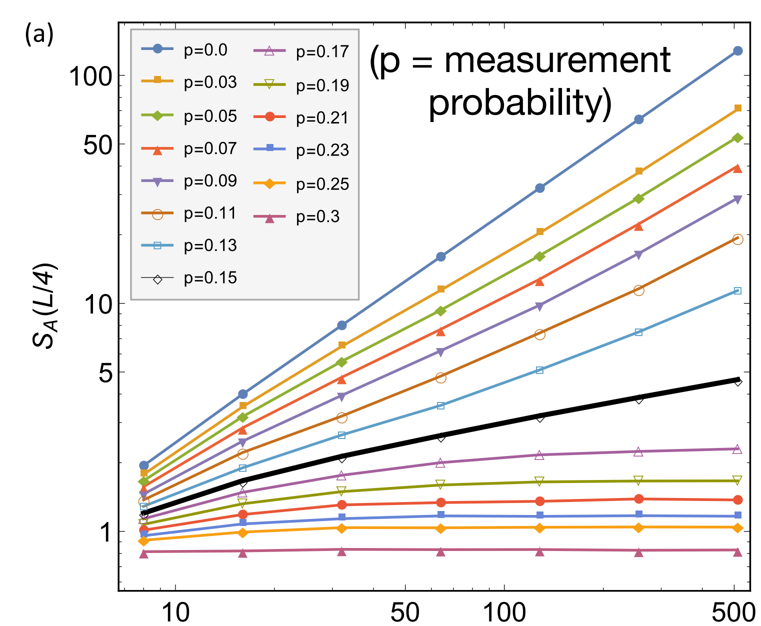
Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

Single-body localization: [Anderson 1958];
Many-body localization: [Basko, Aleiner, Altshuler 2005; Oganesyan, Huse 2007, Pal, Huse 2010, ...]

Another route: Quantum Zeno effect.





[figures from Li, Chen, Fisher 2018]

[Li, Chen, Fisher 2018; Skinner, Ruhman, Nahum 2018; Chan et al 2019; Gullans, Huse, 2019; Bao et al 2019; Jian et al 2019...]

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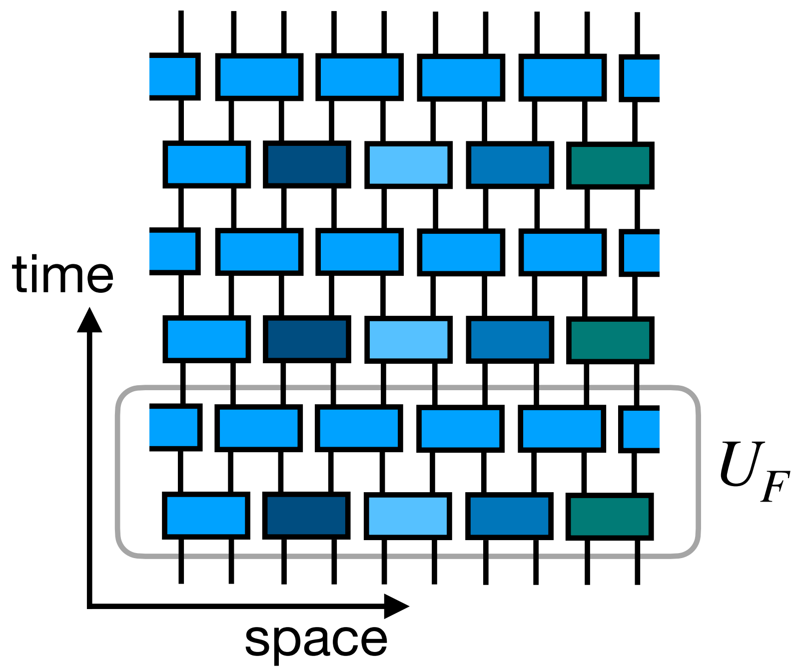

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Single-body localization: [Anderson 1958];
Many-body localization: [Basko, Aleiner, Altshuler 2005; Oganesyan, Huse 2007, Pal, Huse 2010, ...]

A puzzle: Consider time evolution with:

$$H = \int dx \, \left[(\nabla \phi(x))^2 + \Pi^2(x) + m^2(x)\, \phi^2(x) + u(x)\, \phi^4(x) \right]$$

$$[\hat{\phi}(x), \hat{\Pi}(x')] = i\, \delta(x - x')$$

Or, alternatively the following Floquet operator

$$U_F = e^{i \sum_x \hat{\Pi}^2(x)} e^{i \sum_x \left(\hat{\phi}(x+1) - \hat{\phi}(x)\right)^2} e^{i \sum_x V(\hat{\phi}(x), x)}$$

$$[\hat{\phi}(x), \hat{\Pi}(x')] = i\, \delta_{x,x'}$$

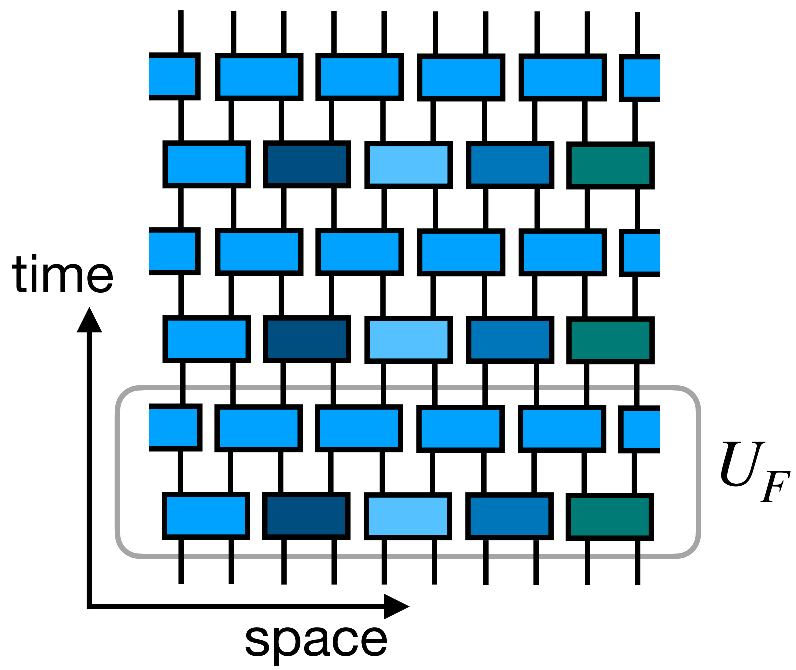
$V(\hat{\phi}(x), x)$ arbitrary function of x, e.g.,

$$V(\hat{\phi}(x), x) = m^2(x)\, \hat{\phi}^2(x) + u(x)\, \hat{\phi}^4(x) + \ldots$$

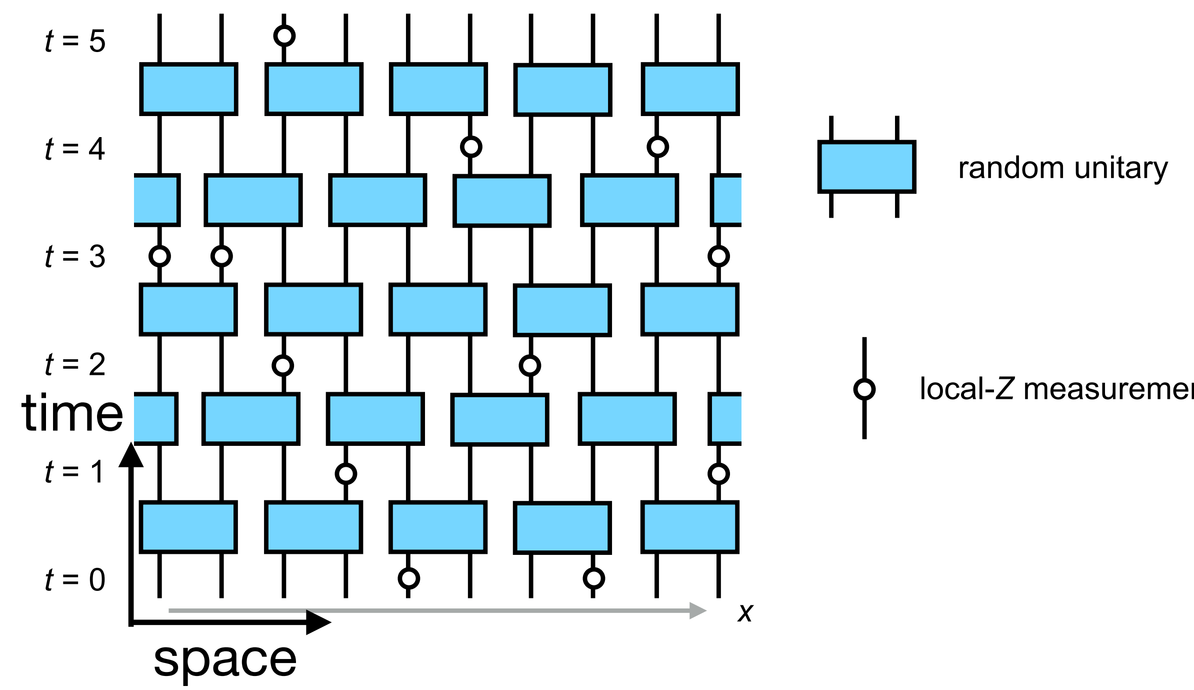
Can either of these systems show many-body localization? What about Bose-Hubbard model which looks somewhat similar?

How to obstruct entanglement growth?

One route: spatial disorder.



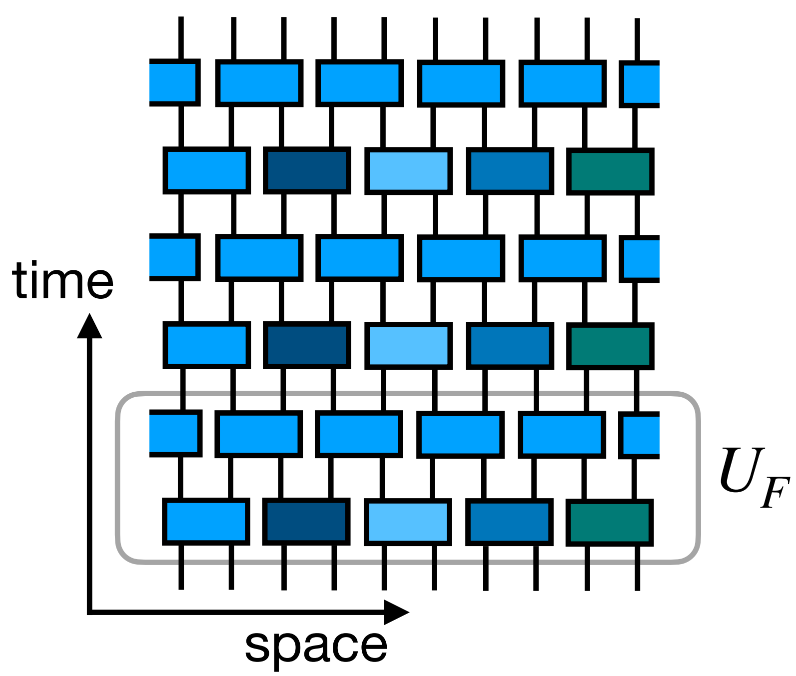
Another route: Quantum Zeno effect.



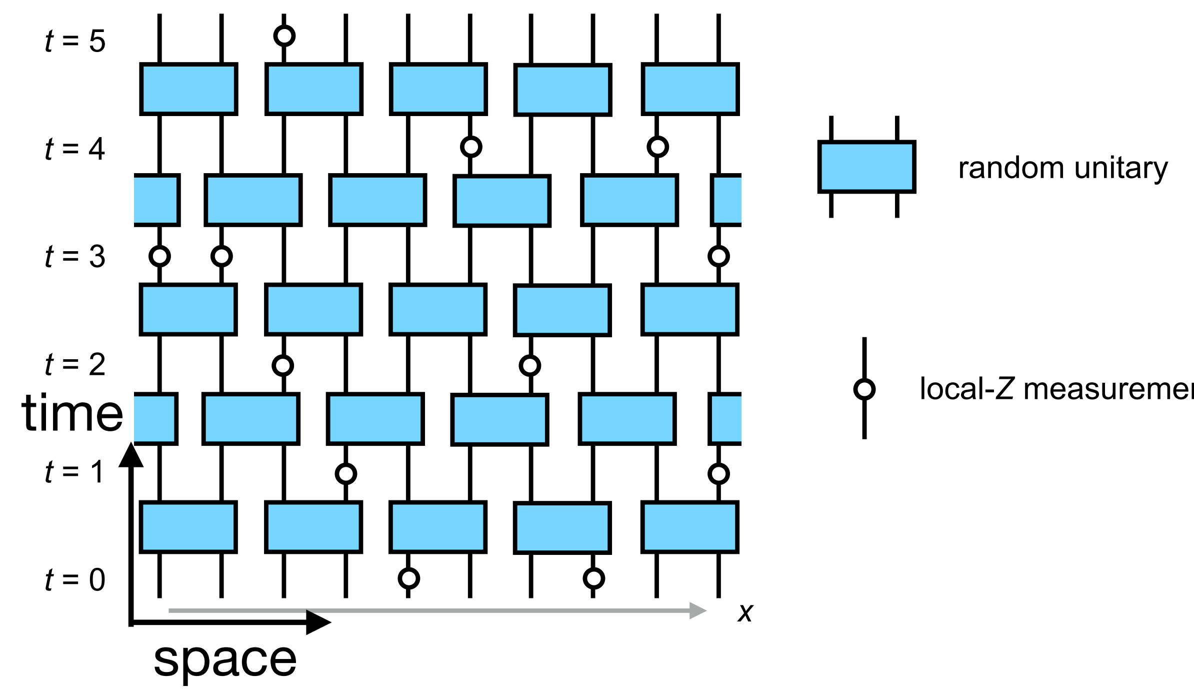
Unitary evolution	Non-unitary evolution
Requires time-translation	Time-translation not required
No post-selection required	Post-selection required

How to obstruct entanglement growth?

One route: spatial disorder.



Another route: Quantum Zeno effect.



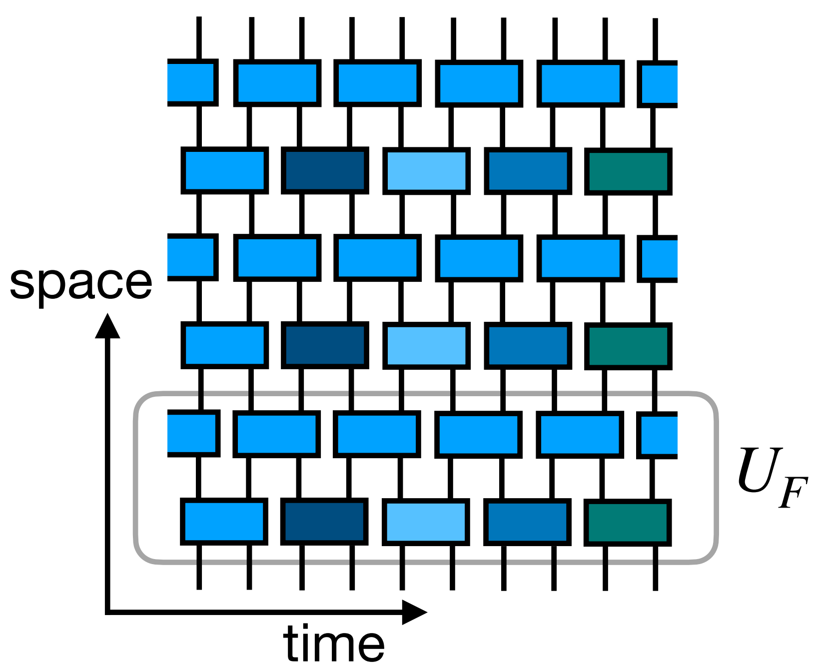
Main result

“Space-time rotation” of a circuit that hosts localization transition yields a new circuit that hosts a Zeno-type transition.

[TG, Tsung-Cheng Lu 2021; Ippoliti, Rakovszky, Khemani 2021]

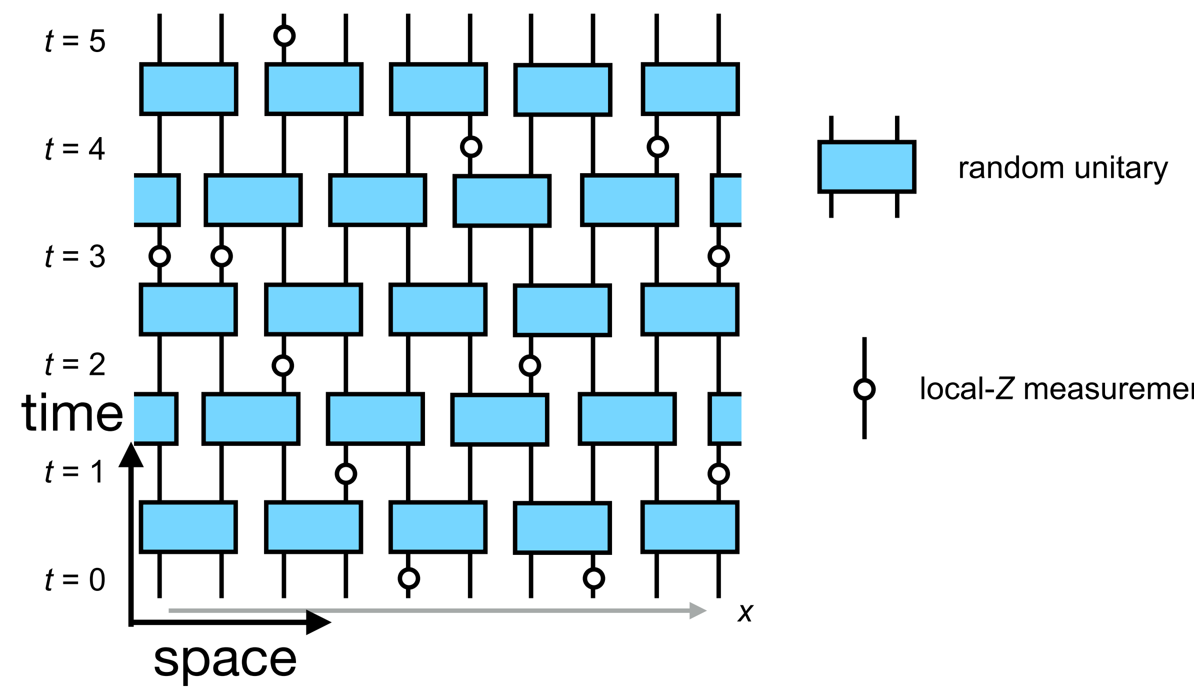
How to obstruct entanglement growth?

One route: spatial disorder.



$\equiv$

Another route: Quantum Zeno effect.



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"Space-time rotation" of a circuit that hosts localization transition yields a new circuit that hosts a Zeno-type transition.

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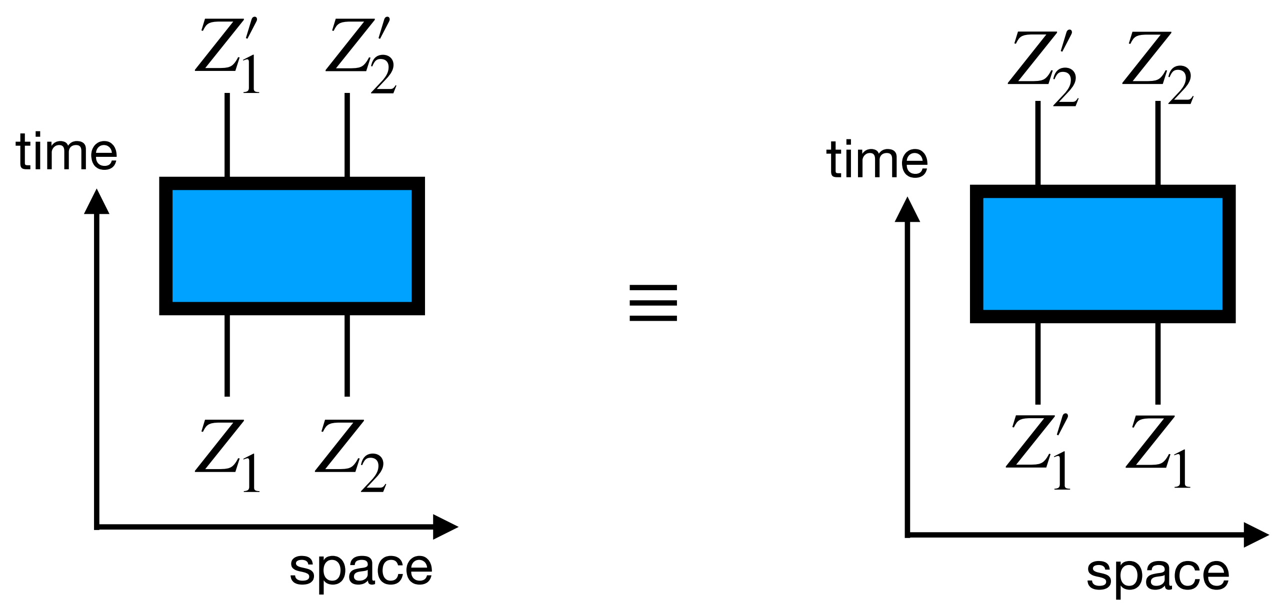
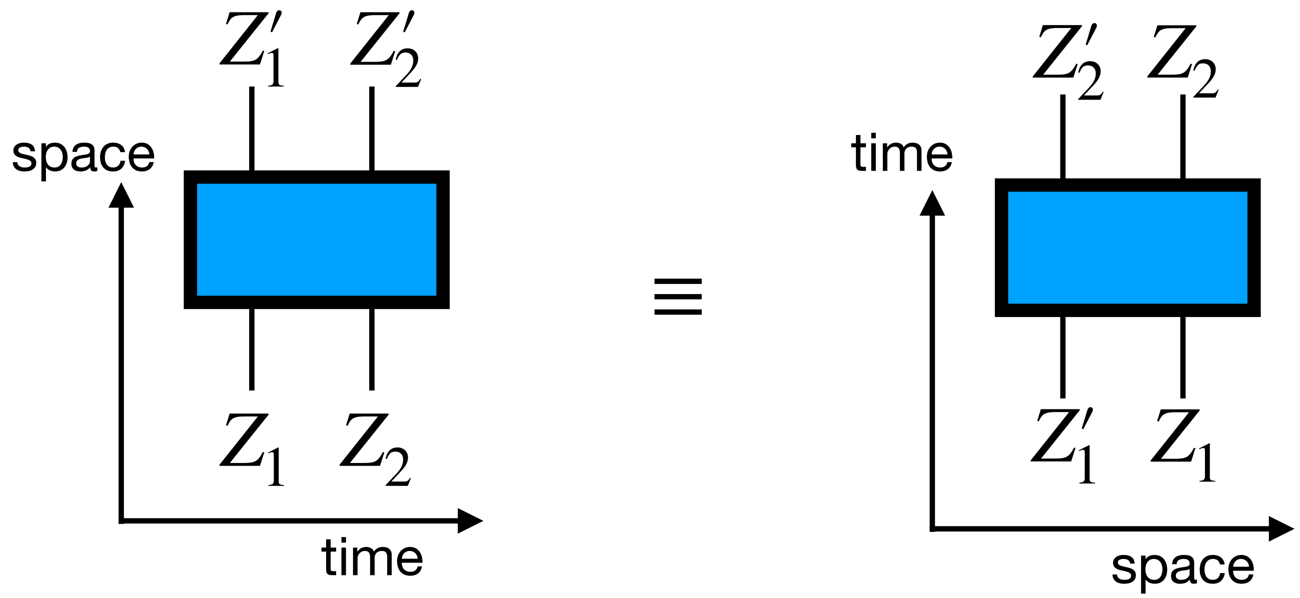
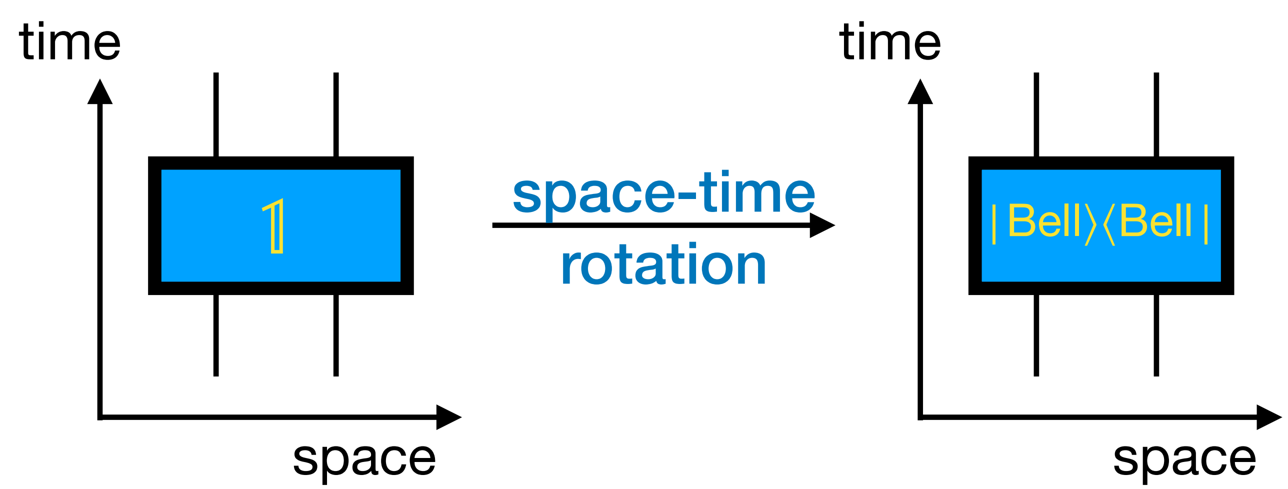
Space-time rotation of a circuit



[Akila, Waltner, Gutkin, Guhr 2016; Bertini, Kos, Prosen 2018;
Napp et al 2020; Ippoliti, Khemani 2020,..., cf. Betsuyaku 1984 (imaginary time)]

Space-time rotation of a circuit

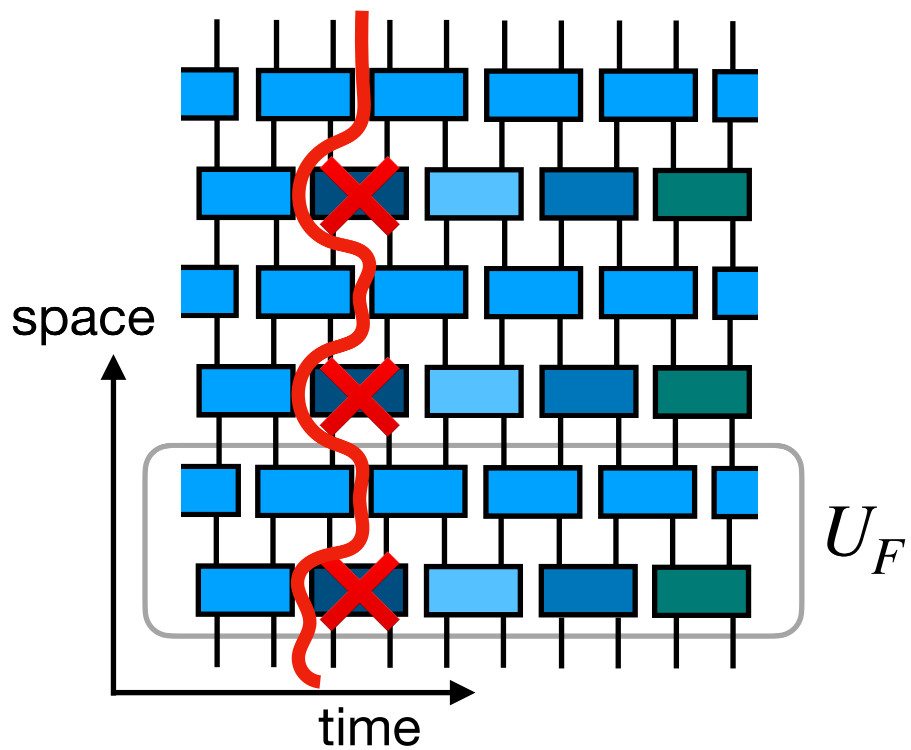


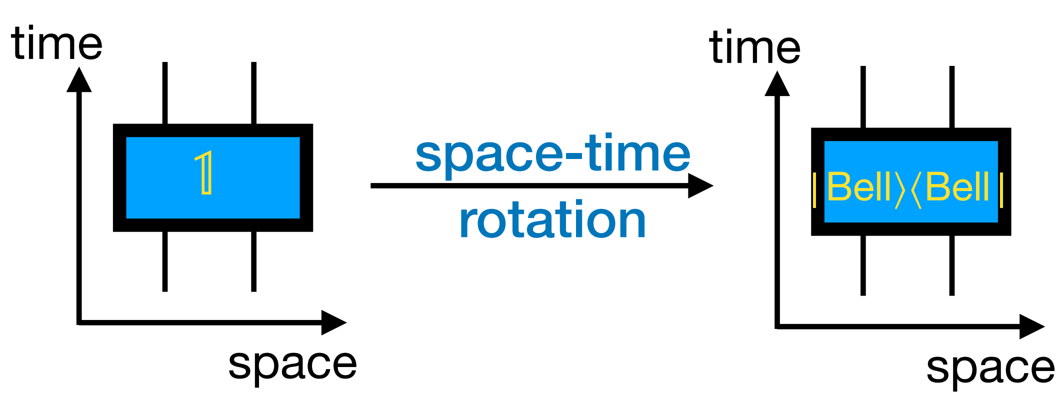


$$|\text{Bell}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

[Akila, Waltner, Gutkin, Guhr 2016; Bertini, Kos, Prosen 2018;
Napp et al 2020; Ippoliti, Khemani 2020,..., cf. Betsuyaku 1984 (imaginary time)]

Consider extreme limit of localization where the system separates into two decoupled regions





Example # 1: Space-time dual of a single-particle localization transition

$$U_F = e^{iJ\sum_r X_r X_{r+1}} e^{i\sum_r h_r Z_r}$$ Unitary circuit = $(U_F)^{\text{time}}$

h_r quasi-periodic, $h_r = 2.5 + \lambda\cos(2\pi Q r + \delta)$ $Q = \frac{2}{1+\sqrt{5}}$

"Floquet Aubry-Andre-Harper" model

Space-time dual also free-fermion circuit, albeit non-hermitian.

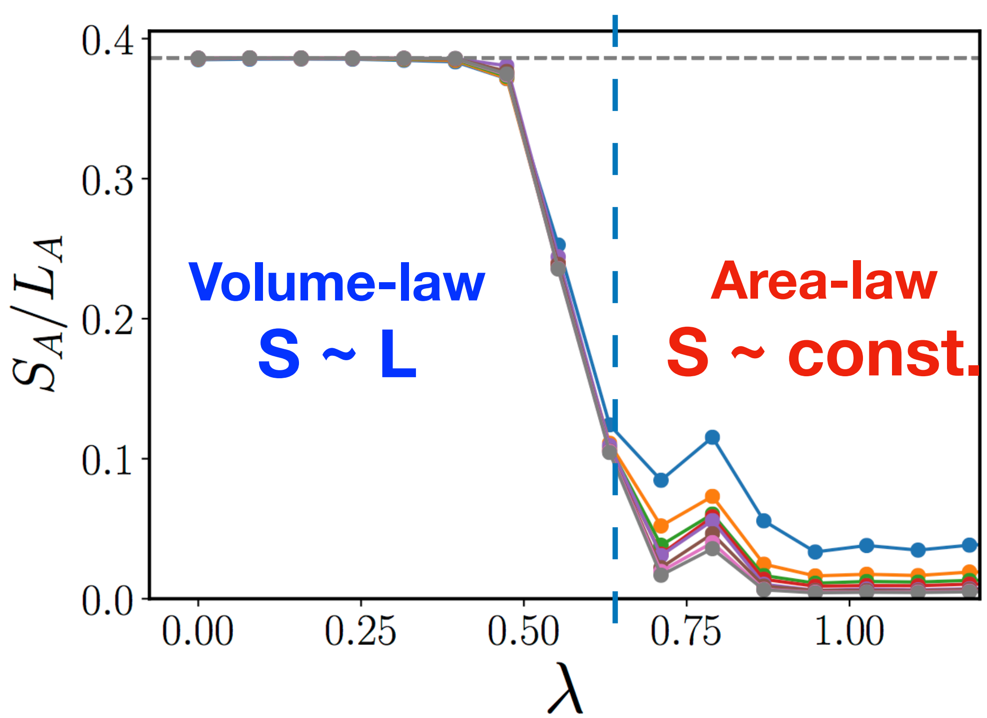
Previous works on free fermion systems shows that one doesn't expect volume law entanglement for generic non-Hermitian Hamiltonians [Jian et al 2020; Chen et al 2020; Tang, Chen, Zhu 2001;...]. For non-unitary circuits consisting of unitary evolution+ only projective measurements, this can be argued fairly rigorously [Cao, Tilloy, De Luca 2019; Fidkowski, Haah, Hastings 2020].

Example # 1: Space-time dual of a single-particle localization transition

Unitary circuit = $(U_F)^{\text{time}}$

$$U_F = e^{iJ\sum_r X_r X_{r+1}} e^{i\sum_r h_r Z_r}$$

$$h_r = 2.5 + \lambda \cos(2\pi Q r + \delta)$$



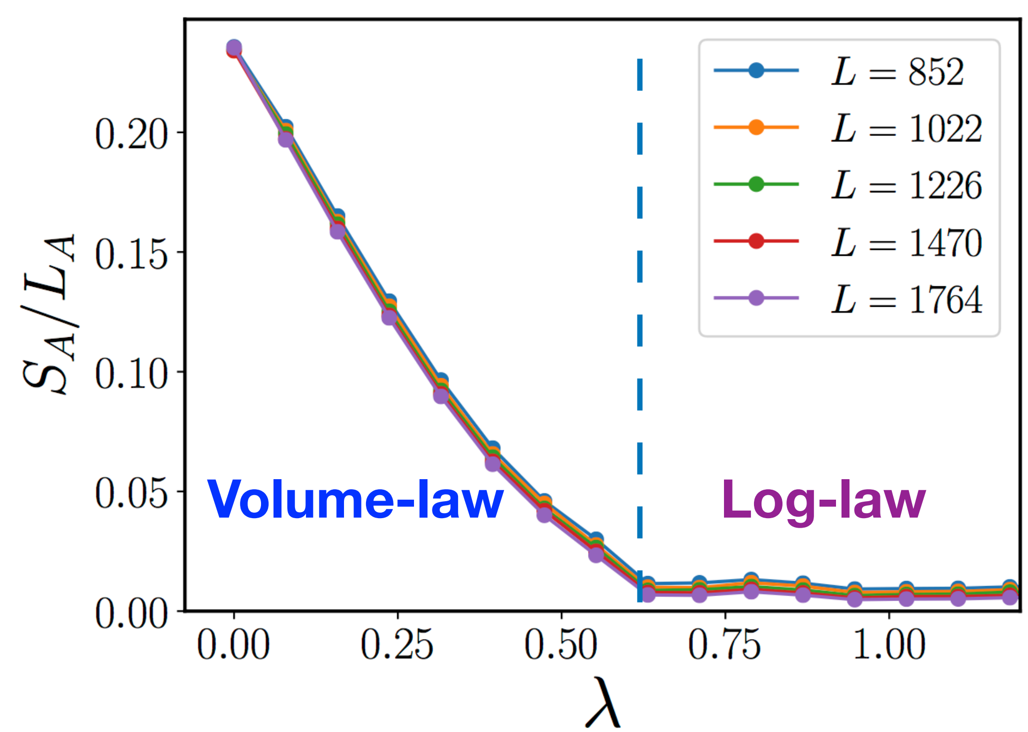
space-time rotation →

$$\lambda_c \approx 0.64$$

Non-unitary circuit

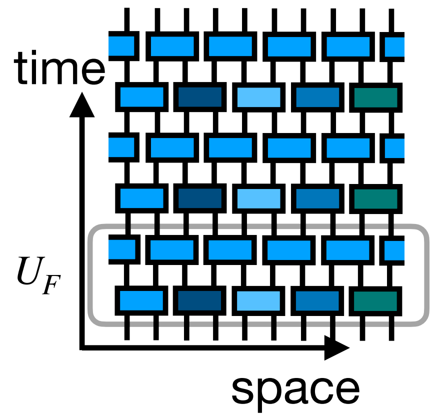
$$V(T) = \prod_{t=1}^{T} e^{i\tilde{h}\sum_r Z_r} e^{i\sum_r \tilde{J}(t) X_r X_{r+1}}.$$

$$|Re(\tilde{h})| = |Re(\tilde{J})| = \pi/4$$



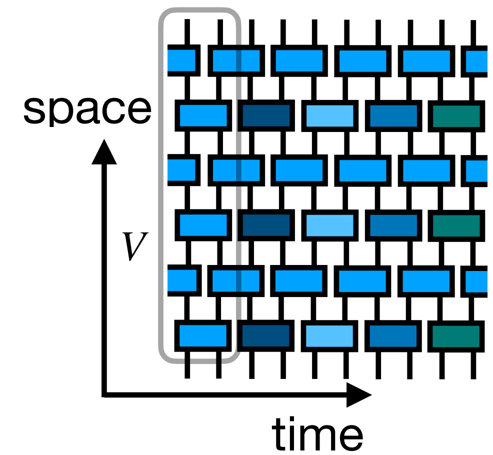
[TG, Tsung-Cheng Lu 2021]

Basic Intuition



$$U_F = e^{iJ\sum_r X_r X_{r+1}} e^{i\sum_r h_r Z_r}$$

$$h_r = 2.5 + \lambda\cos(2\pi Q r + \delta)$$



$$V(T) = \prod_{t=1}^{T} e^{i\tilde{h}\sum_r Z_r} e^{i\sum_r \tilde{J}(t) X_r X_{r+1}}.$$

$$\tilde{J}(t) = -\pi/4 + \frac{i}{2}\log(\tan h_t)$$

$$\tilde{h} = \tan^{-1}(-ie^{-2iJ})$$

When h_t for *some* t becomes π, the non-unitary circuit at that time-slice corresponds to a pure projector. In the original unitary circuit, this condition corresponds to vanishing of Jordan-Wigner Majorana hopping on *some* bond.

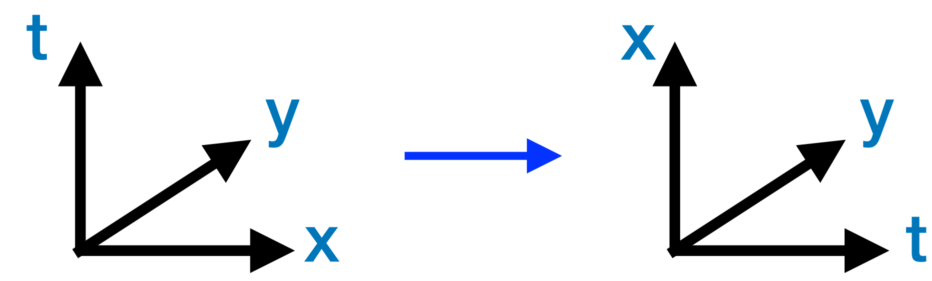
$$\Rightarrow \lambda_c = \pi - 2.5 \approx 0.64$$

Example # 2: Space-time dual of a unitary 2+1D circuit

$$U_F = e^{-i\frac{\pi}{4}\sum_{\langle ij\rangle} J_{ij} Z_i Z_j} e^{-i\frac{\pi}{4}\sum_i h_i X_i}$$

independent random variables $J_{ij}, h_i = 0, 1$

Probability p, 1-p



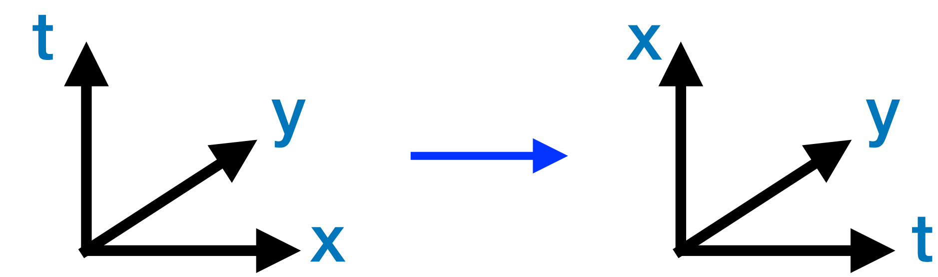
The space-time rotated non-unitary circuit consists of only unitaries and forced measurements.

Both rotated and unrotated circuits can be simulated efficiently since they consist of Clifford gates.

[TG, Tsung-Cheng Lu 2021]

Example # 2: Space-time dual of a unitary 2+1D circuit

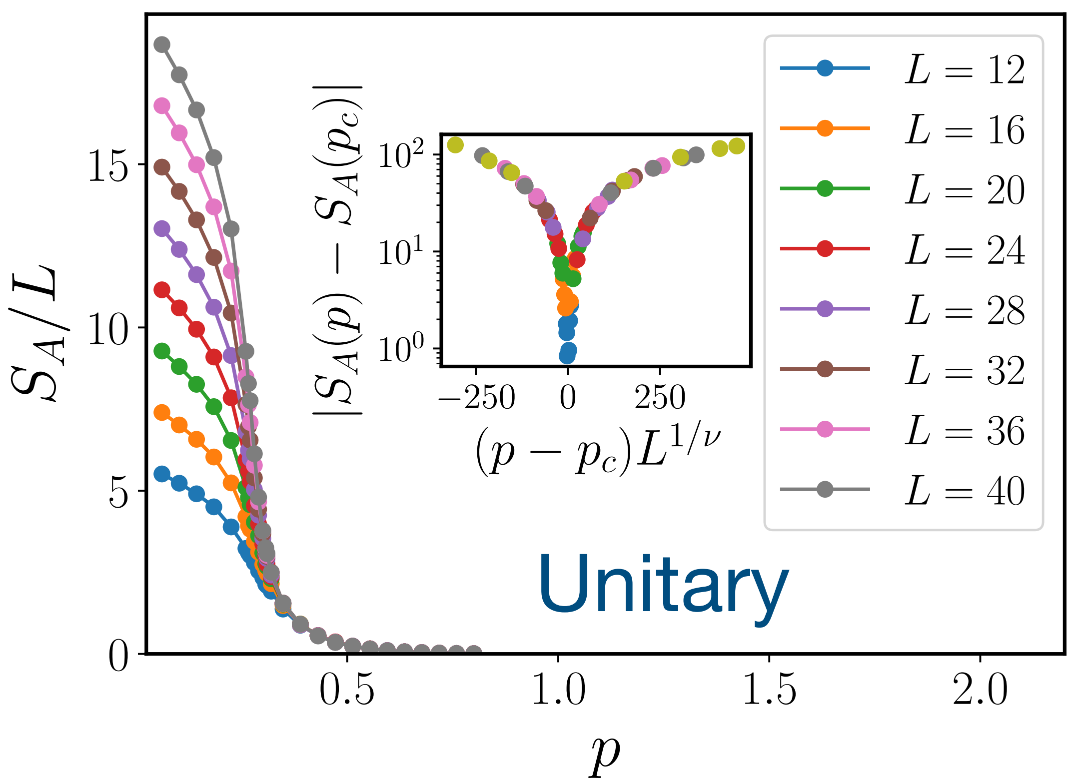
$$U_F = e^{-i\frac{\pi}{4}\sum_{\langle ij\rangle} J_{ij} Z_i Z_j}\, e^{-i\frac{\pi}{4}\sum_i h_i X_i}$$



independent random variables $J_{ij}, h_i = 0, 1$

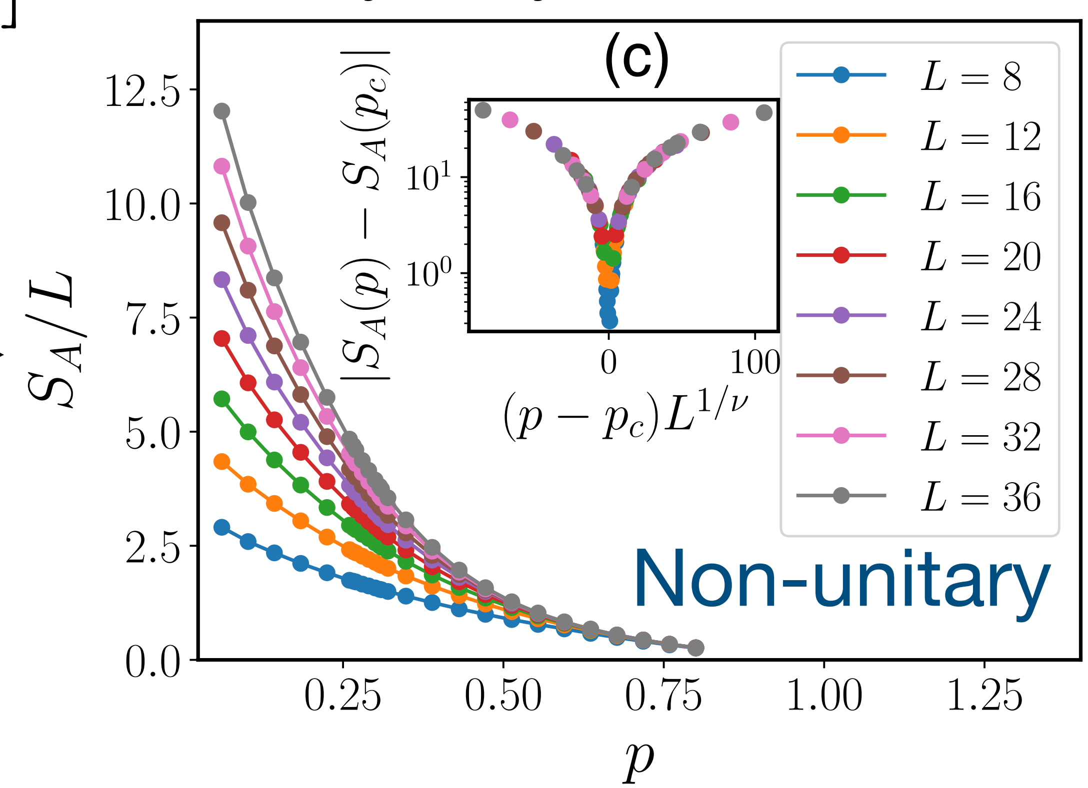
Probability p, 1-p

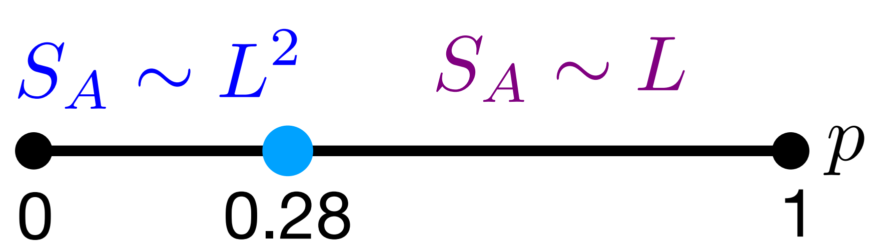
"Floquet Clifford Localization transition" [Chandran, Laumann 2015]



space-time rotation

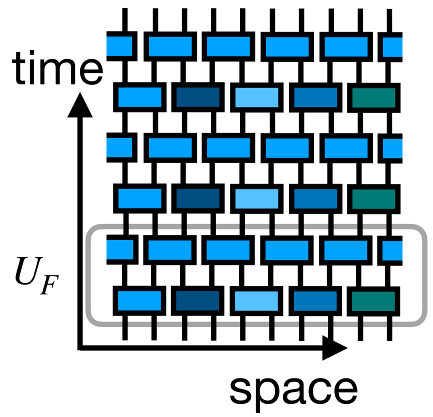
"Many-body Zeno transition"

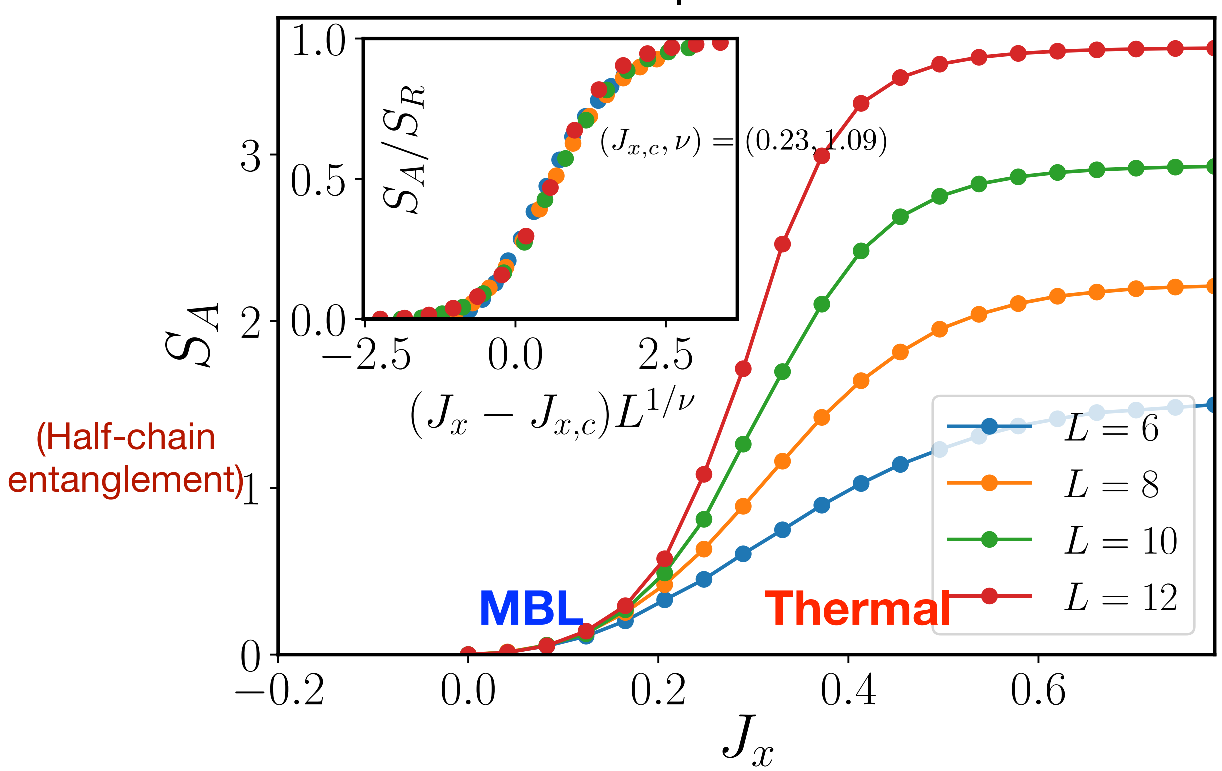




[TG, Tsung-Cheng Lu 2021]

Example # 3: Rotation of Floquet-MBL circuit

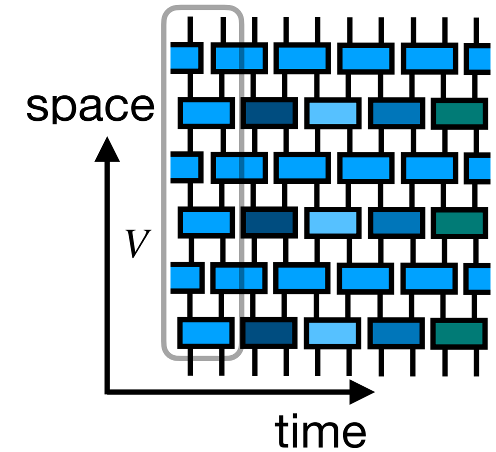


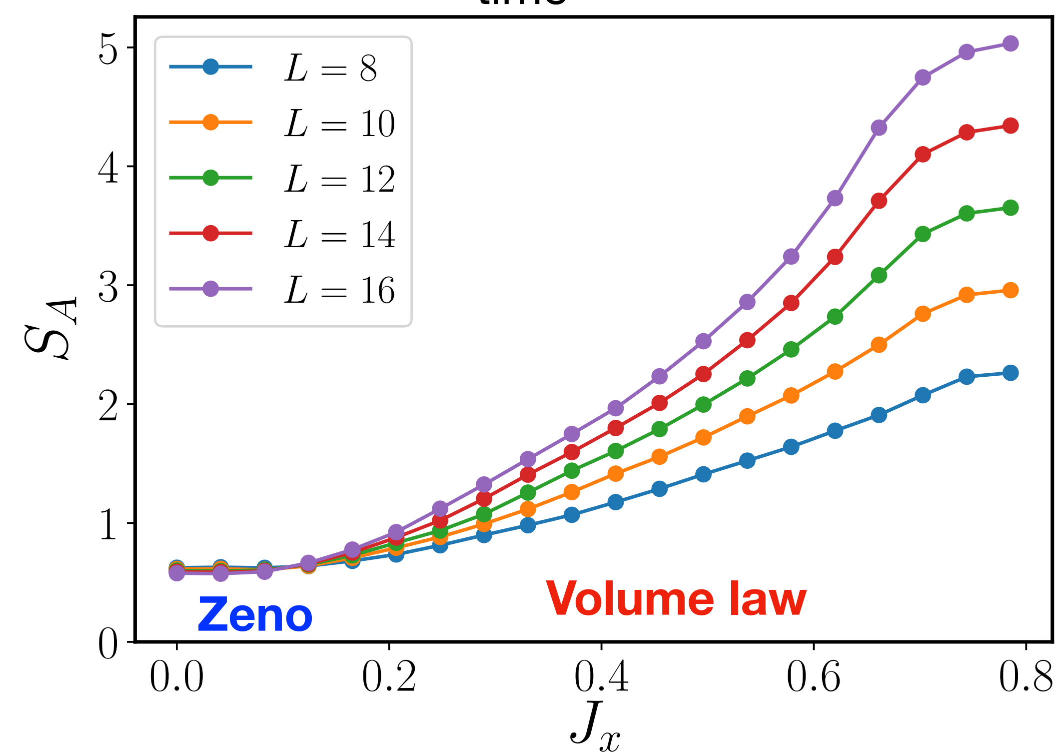


$$U_F = e^{iJ_x \sum_r X_r} e^{-i\tau \sum_r Z_r Z_{r+1} - i\tau \sum_r h_r Z_r}$$

$\tau = 0.8$, h_r : random Gaussian variable

[Ponte et al 2015; Abanin, De Roeck, Huveneers 2016; Zhang, Khemani, Huse 2016]





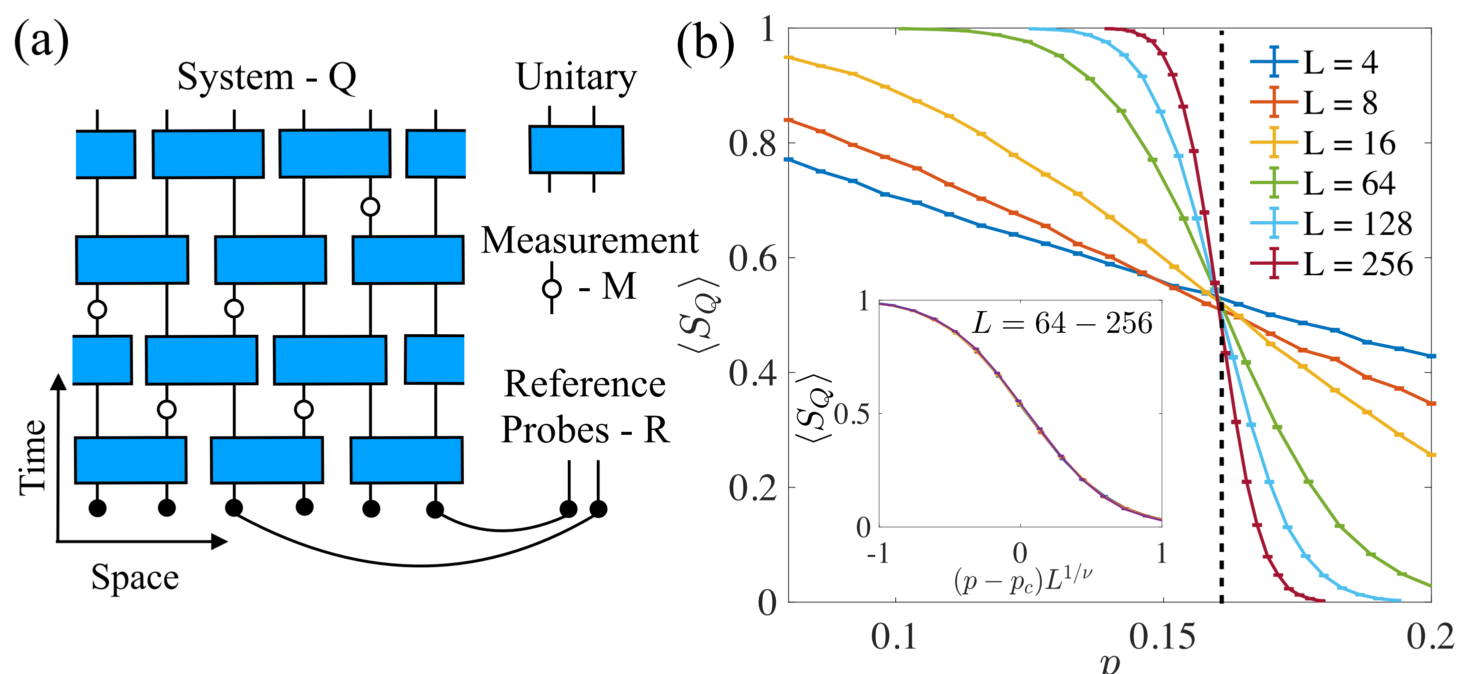
$$V(T) = \prod_{t=1}^{T} V_t, \quad V_t = e^{i\tilde{J}_x \sum_r X_r} e^{i\tilde{J}_z \sum_r Z_r Z_{r+1} - i\tau h(t) \sum_r Z_r}$$

Translationally invariant in space
Disordered in time

Probing Zeno transition using purification dynamics of a reference-qubit

Gullans, Huse (2019) proposed coupling a reference-qubit to the system of interest, and studying its long-time entanglement.

From Gullans, Huse (2019)



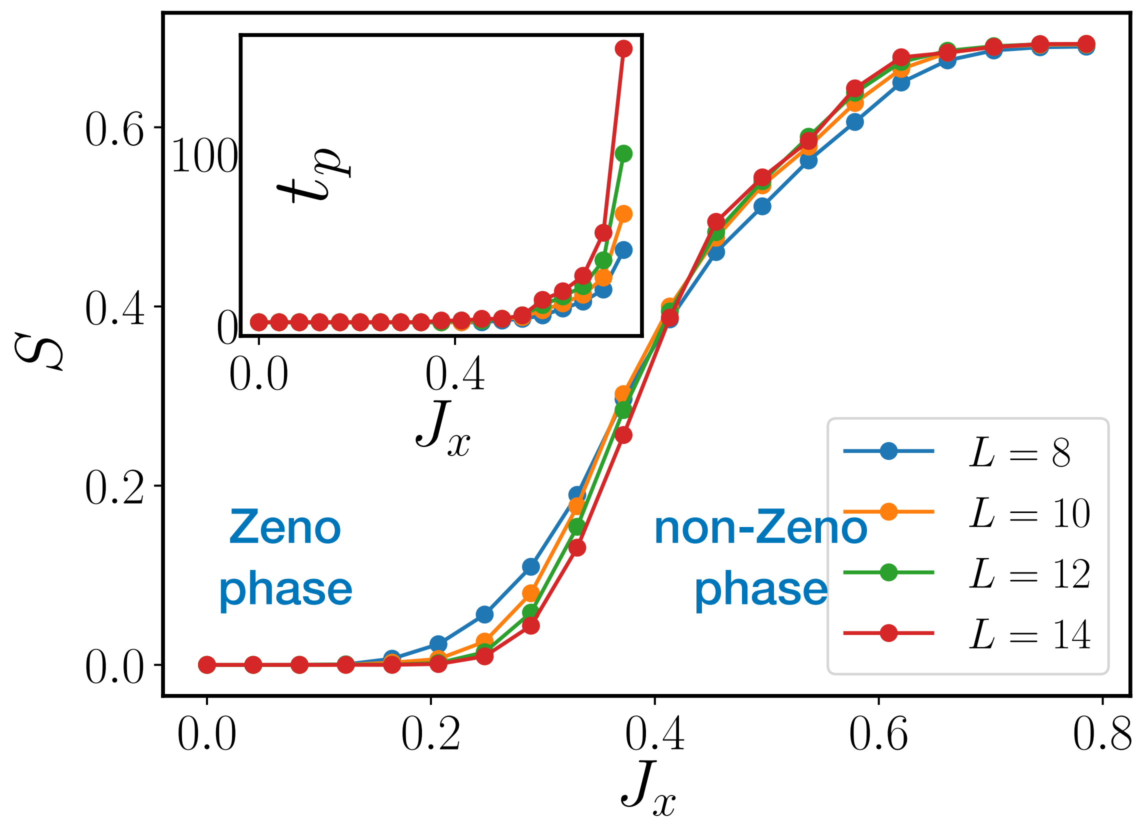
non-Zeno phase

Zeno phase

In the Zeno phase, the qubit purifies itself in O(1) time.

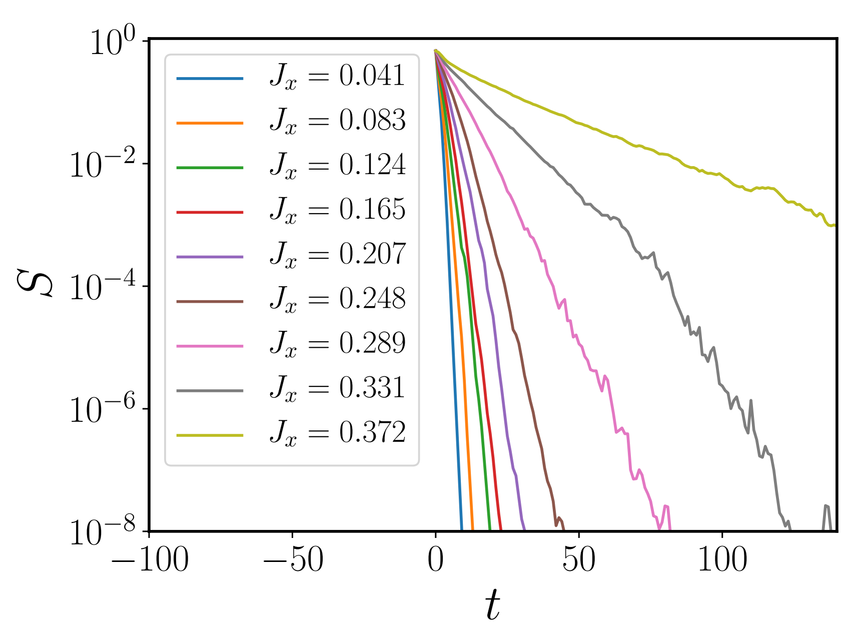
In the non-Zeno phase, the purification time is exponential in system size.

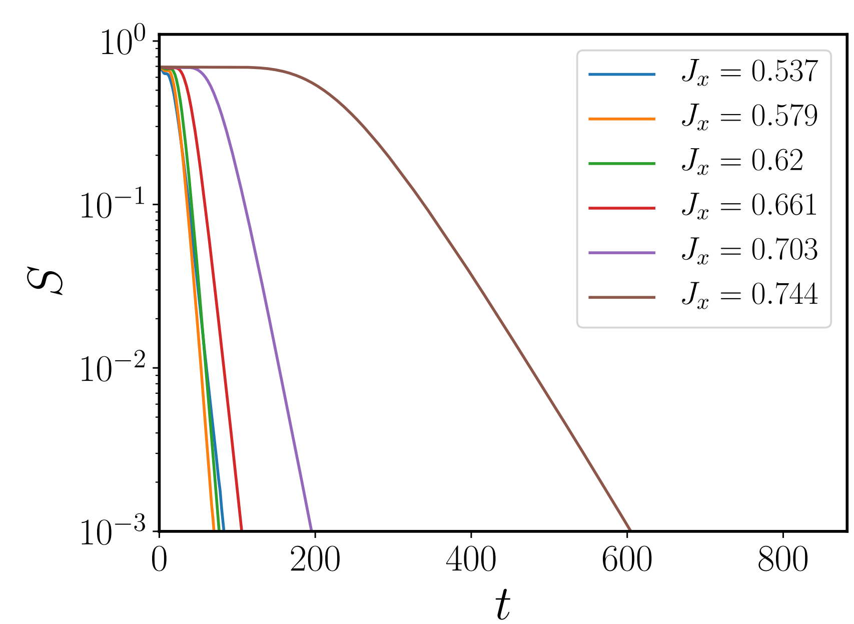
Purification dynamics in the space-time dual of Floquet MBL



Inset: Purification time = O(1) in the Zeno phase. Super-linear in the non-Zeno (“error-correcting”) phase

Time dependence of reference-qubit entanglement





Implications for shallow circuits?

Efficient classical simulation of random shallow 2D quantum circuits

John Napp* Rolando L. La Placa† Alexander M. Dalzell‡
Fernando G. S. L. Brandão§ Aram W. Harrow¶

March 10, 2020

It was shown that a class of shallow circuits with gates chosen from a certain random distribution can be efficiently simulated classically via space-time rotation. In the regime of classical simulatability, the rotated circuit was argued to have area-law EE. One may wonder if this result generalizes to other circuits/Hamiltonians…

A couple conjectures:

1. Time evolution of translationally invariant, shallow, chaotic circuits typically cannot be simulated using polynomial resources (the rotated circuit will likely have volume law entanglement). Consistent with [Bermejo-Vega et al 2017].

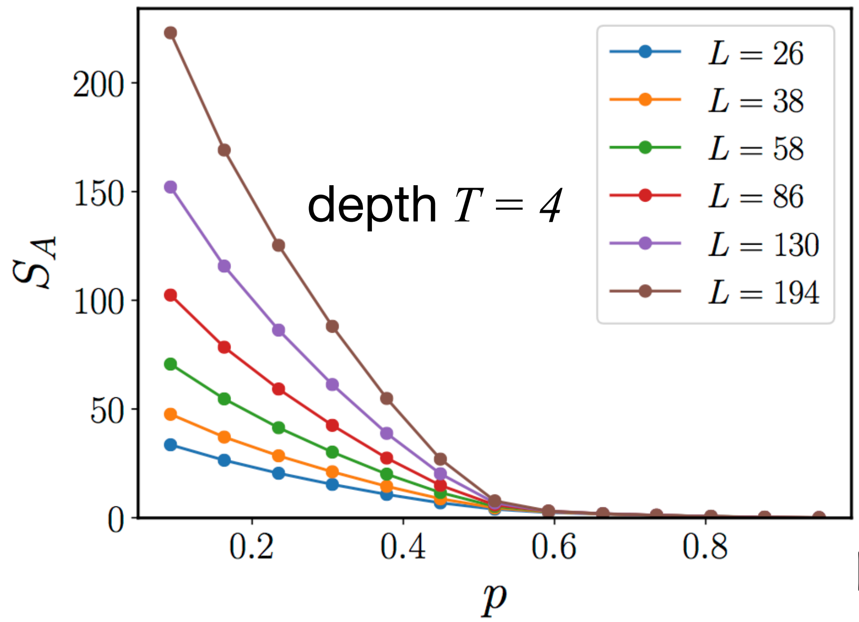
2. Tuning randomness in a 2d shallow circuit/Hamiltonian can sometimes drive a transition between area to volume law in the rotated non-unitary circuit. How generic is this? (is Clifford circuit too special for this feature?)

Implications for shallow circuits?

$$U = \prod_{t=1}^{T} \left[e^{-i\pi/4 \sum_{\langle r,r' \rangle} J(r,r',t)\, Z(r)\, Z(r')} e^{-i\pi/4 \sum_r h(r,t)\, X(r)} \right]$$

(space-time randomness)

Disorder tuned transition in rotated non-unitary circuit

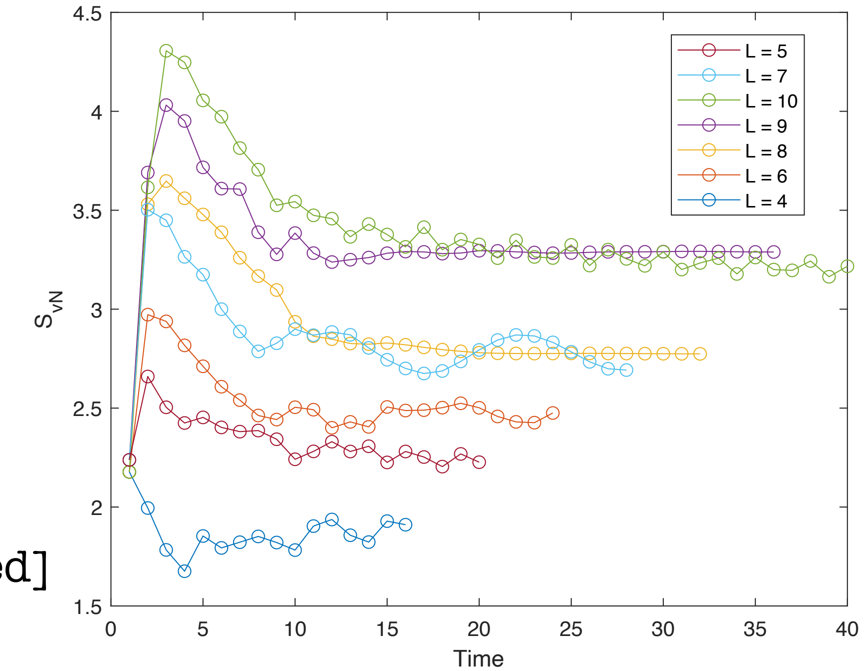


[Lu, TG, unpublished]

$$U = e^{-i \sum_{\langle r,r' \rangle} Z(r)\, Z(r')} e^{-ih \sum_r X(r)}$$

(translationally invariant)

Entanglement barrier in rotated non-unitary circuit



A couple conjectures:

1. Time evolution of translationally invariant, shallow, chaotic circuits likely cannot be simulated using polynomial resources (the rotated circuit will likely have volume law entanglement). Consistent with [Bermejo-Vega et al 2017].

2. Tuning randomness in a 2d shallow circuit/Hamiltonian can sometimes drive a transition between area to volume law in the rotated non-unitary circuit. How generic is this? (is Clifford circuit too special for this feature?)

Summary and a few questions…

- Topological negativity seemingly a candidate order parameter for finite-T topological order.
- Mixed-state entanglement provides a sharper distinction between classical and quantum phase transitions.
- Space-time rotation provides a connection between two distinct mechanisms of entanglement obstruction in a class of circuits (localization ↔ Zeno).
- "Industrial" applications of negativity? e.g., optimize parameters in a noisy quantum computer by maximizing negativity between qubits.
- Are there models where singularity exists *only* in quantum correlations at finite temperature? "Truly quantum finite-T phase transitions".
- Calculation of mixed-state entanglement measures other than negativity?

A puzzle: Consider coupled oscillators…

$$U_F = e^{i\sum_x \hat{P}^2(x)} e^{i\sum_x \left(\hat{\phi}(x+1) - \hat{\phi}(x)\right)^2} e^{i\sum_x V(\hat{\phi}(x),x)}$$

$[\hat{\phi}(x), \hat{P}(x')] = i\delta_{x,x'}$ $V(\hat{\phi}(x),x)$ arbitrary function of x, e.g.,

$$V(\hat{\phi}(x),x) = m^2(x)\,\hat{\phi}^2(x) + u(x)\,\hat{\phi}^4(x) + \ldots$$

Can this circuit show many-body localization?

Most likely not, because the space-time rotated circuit is unitary, and random along the time direction.

Similar to **Exact Spectral Form Factor in a Minimal Model of Many-Body Quantum Chaos**

Bruno Bertini, Pavel Kos, and Tomaž Prosen

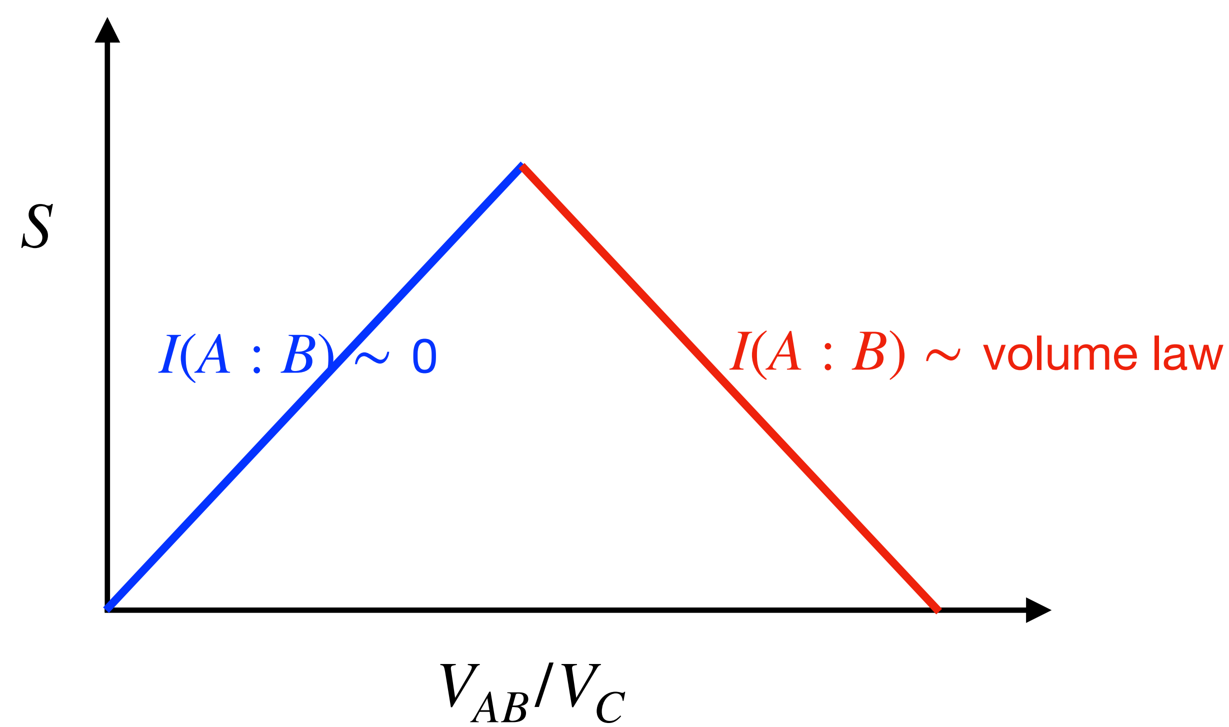
(2018)

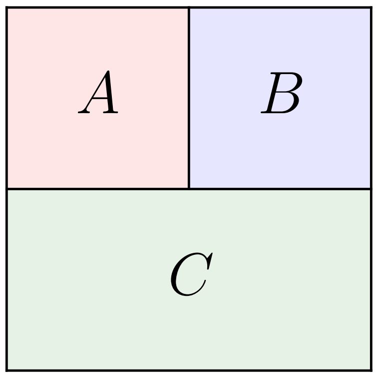
Is it exactly solvable for arbitrary potentials $V(\hat{\phi}(x),x)$?

Detecting absence of thermalization using mixed-state entanglement

When a system does not thermalize, expectation that the entanglement between two subsystems A, B must be “large”.

Trivial example: random Haar state

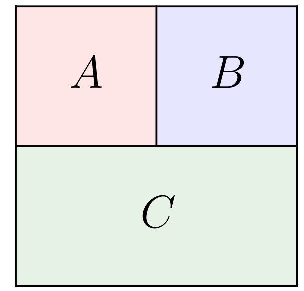




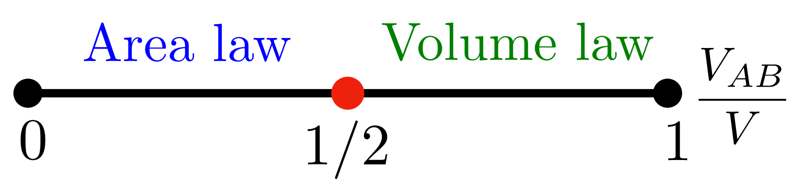
How to quantify this without polluting with classical correlations?

Implications for localized, integrable and scar states?

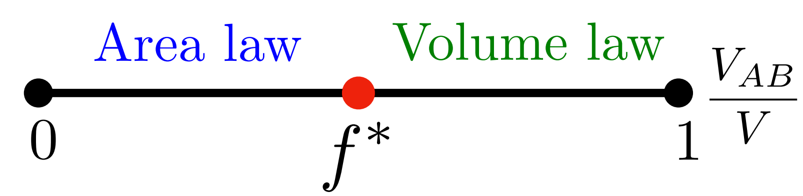
Summary of Results



Non-integrable
eigenstates



Non-integrable
long-time states



Integrable
eigenstates

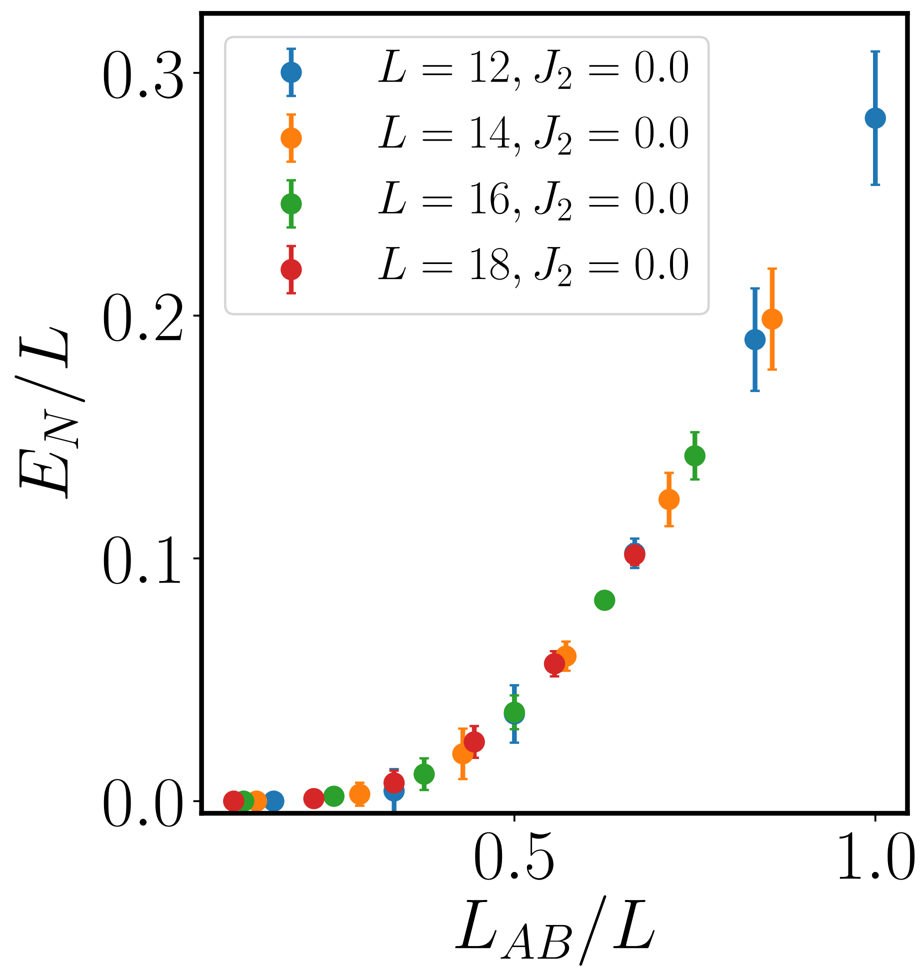
Integrable
long-time states

MBL
eigenstates

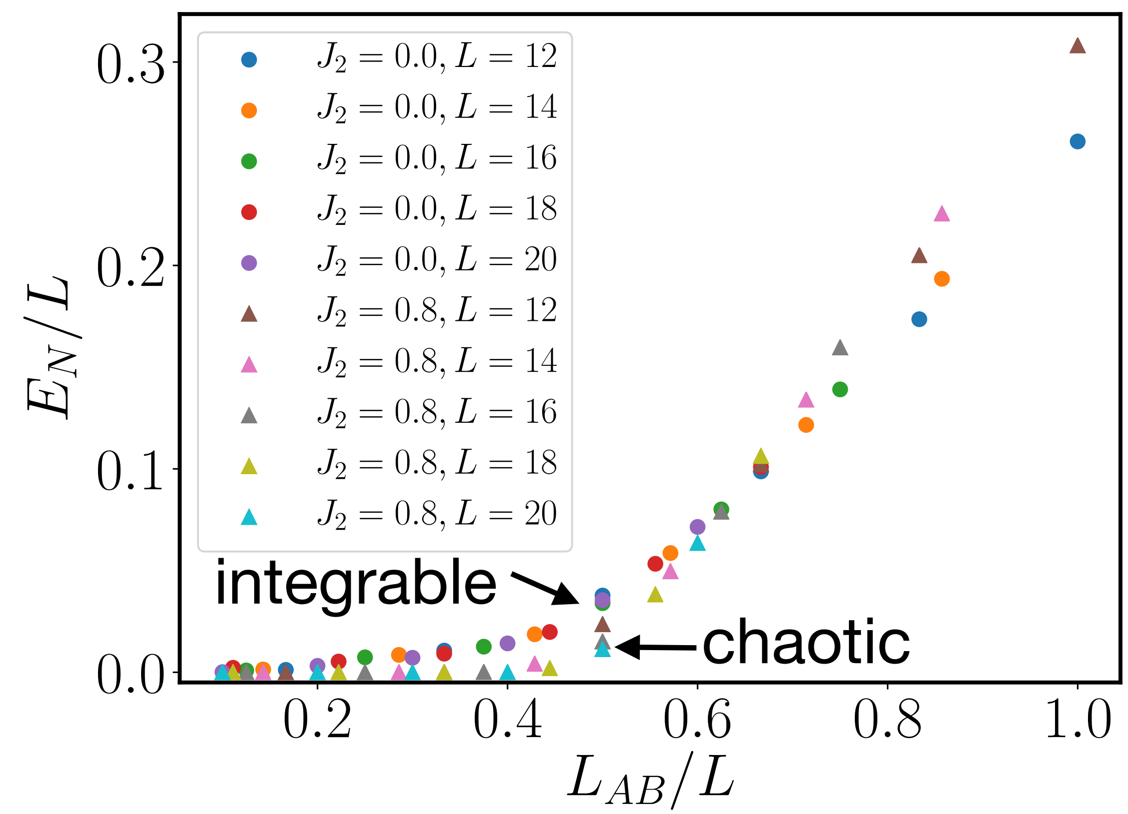
MBL
long-time states

[Lu, Grover 2020]

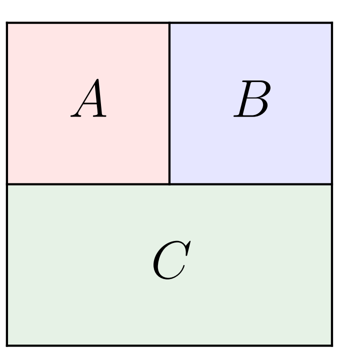
Volume-law negativity in integrable systems



Eigenstates

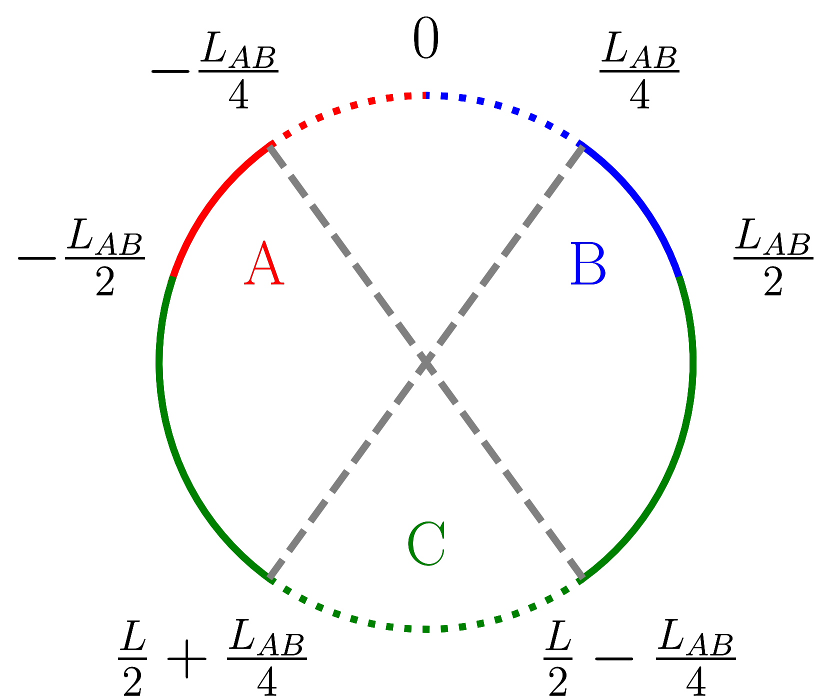


Time-evolved states



Quasiparticle picture for integrable systems

A = red,
B = blue,
C = green.



For quasiparticles to generate entanglement between A,B, one of them belongs to A, and the other to B.

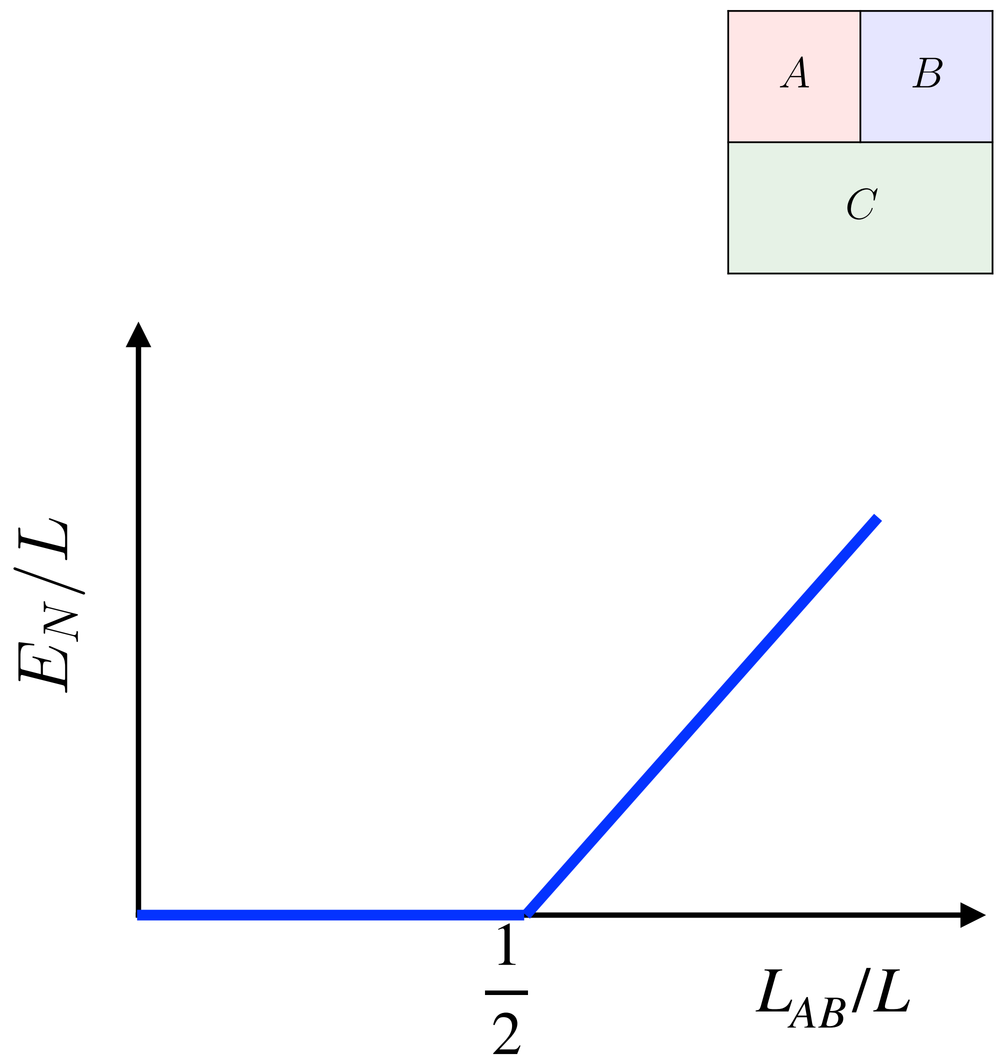
# of quasiparticles that A and B can share $\propto L_{AB}$

Fraction of time these quasiparticles can entangle $\propto L_{AB}/L$

$\Rightarrow$ Time averaged entanglement $\sim \left(L_{AB}/L\right) L_{AB}$ = volume law

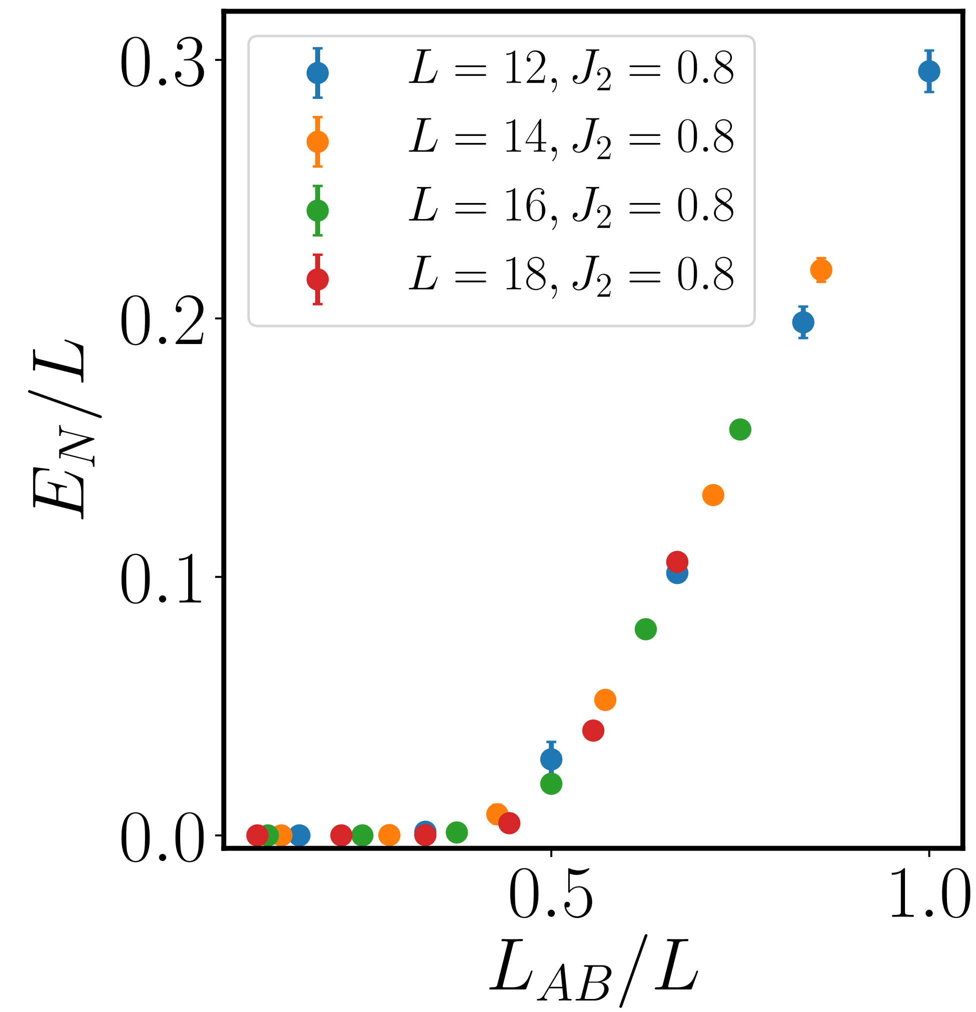
[Cardy, Calabrese 2005]

Negativity transition in a chaotic system



Random Haar state

[Auburn 2012; Bhosale, Tomsovic, Lakshminarayan 2012; Shapourian, Liu, Kudler-Flam, Vishwanath 2020]



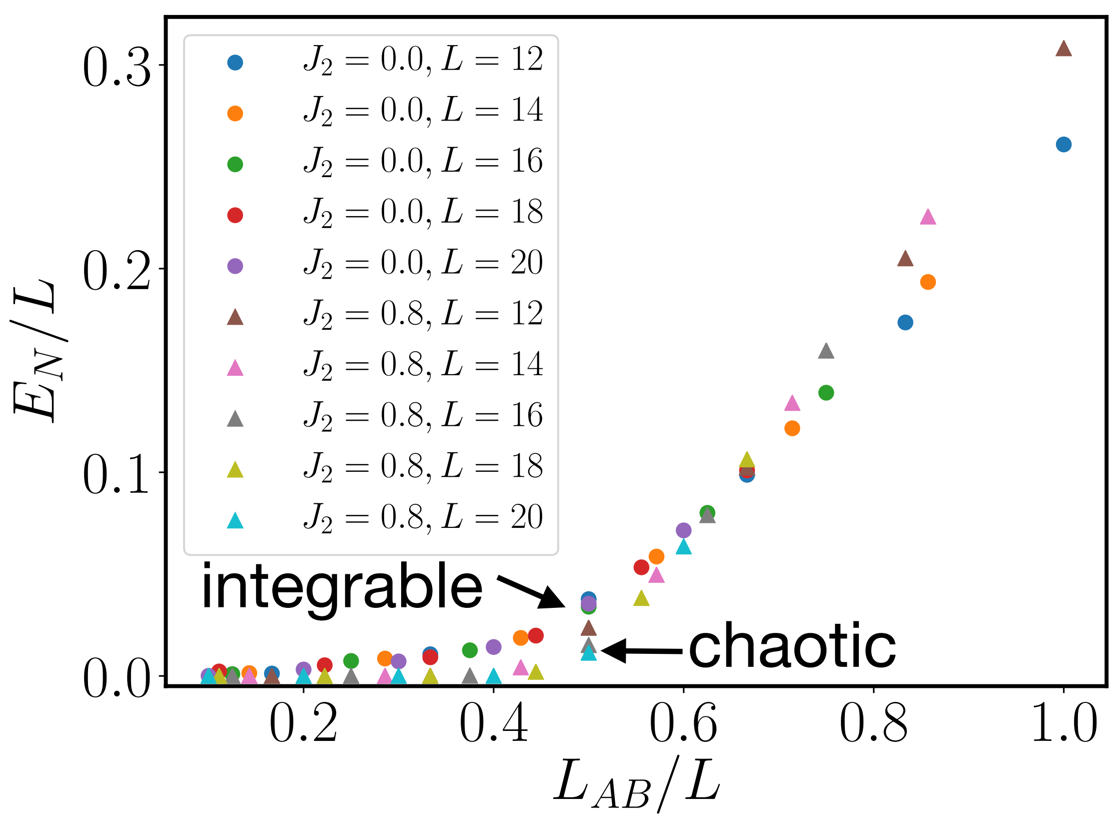
Chaotic eigenstate

$$H = \sum_{i=1}^{L} \left(J_1 \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 S_i^z S_{i+2}^z \right)$$

Negativity transition as a probe of chaos

Integrable Vs Chaotic

$$H = \sum_{i=1}^{L} \left(J_1 \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 S_i^z S_{i+2}^z \right)$$



Chaotic Vs Many-body localized

$$H = \sum_{i=1}^{L} \left(J_1 \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 S_i^z S_{i+1}^z - h_i S_{i+1}^z \right)$$